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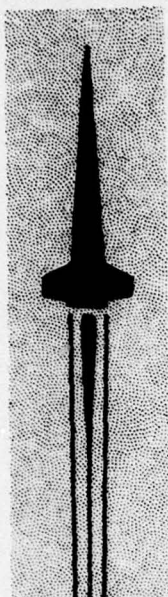
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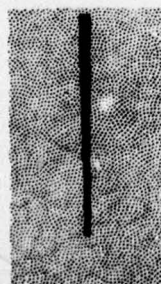


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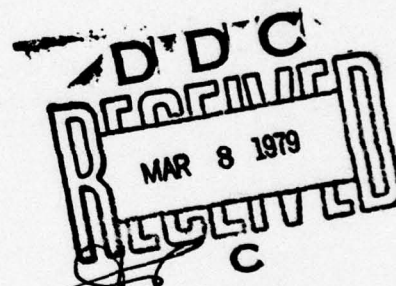
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TECHNICAL REPORT T-CR-79-2

ATMOSPHERIC EFFECTS ON IMAGING SEEKERS

L. J. Pinson
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JULY 1978



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Prepared for
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The major body of knowledge about the effects of the atmosphere on optical radiation is best described by a small number of primary references. One goal of this effort was to establish a list of these primary references which are in the bibliography. Further, specific atmospheric effects are presented in summary form. An alternative objective was to identify deficiencies in the theoretical development as related specifically to imaging seeker applications. This was accomplished through comparison of assumptions and approximation made in the theoretical development with expected conditions for the seeker.			

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ABSTRACT (Concluded)

→ A detailed theoretical development based on Maxwell's equations in a slightly inhomogeneous, weakly absorbing medium is given which describes the optical properties of the atmosphere. From the resultant atmospheric Green's function, the propagation of complex spatial optical amplitude functions from the object to the image plane of a simple lens is given. The atmosphere-lens modulation transfer function (MTF) is then computed from the spatial Fourier transform of the optical intensity function in the image plane.

Contrast in the image plane is computed in terms of the definitions for universal and modulation contrast using the image plane intensity and adding a term for ambient irradiance. The theory as developed gives a rigorous treatment of the contrast transfer and MTF of an atmosphere-lens imaging system for the monochromatic case. It remains to extend the model to give contrast and MTF for the broadband case.

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I. INTRODUCTION

The Advanced Sensors Directorate, U.S. Army Missile Research and Development Command (MIRADCOM), has conducted an extensive exploratory development program in imaging seekers for application on terminal homing missiles. Several prototype seekers have been produced featuring a variety of sensors, detector arrays, and scanning mechanisms. However, the ability to characterize the effects of the atmosphere on this class of seekers is not well established, consequently, predicting performance of a terminal homing weapon using this seeker under a variety of meteorological conditions has not been possible.

The general objective for this effort was to develop the analytical description of the atmospheric effects on an imaging seeker that allows seeker performance to be related to the specific meteorological conditions over the line of sight path from the seeker to the target. Inclusion of detailed atmospheric effects in the parametric analysis of sensor performance produces added complexity which is not in every case separable from effects such as the modulation transfer function (MTF) of the imaging optics. Further, the theoretical development for absorption and scattering effects is provided quite distinctly from that for the effects of refractive index fluctuation. Contrast reduction due to scattering and absorption is based on thermodynamic arguments and, if certain assumptions are accepted, is derived in a form which is readily compared with measured meteorological parameters. On the other hand, the effects of atmospheric turbulence, or refractive index fluctuation, is based on one

of several approximate solutions to Maxwell's equations and results in an expression dependent on an ensemble-averaged "wave structure function" for the atmospheric path.

It was desired to develop an analytical description of the effects of the atmosphere on imaging seekers which includes scattering, absorption and refraction effects in a single theoretical development. Specifically, the theory should show the effects of target size on contrast transfer through the atmosphere, include spectral dependence with an analysis of the expected effects of finite spectral band imaging and provide the basis for the development of an atmospheric measurements program.

The major body of knowledge about the effects of the atmosphere on optical radiation is best described by a small number of primary references. One goal of the current effort was to establish a list of these primary references and to present in easily readable summary form, the specific atmospheric effects described. An alternative objective was to identify deficiencies in the theoretical development as related specifically to imaging seeker applications. This was accomplished through comparison of assumptions and approximations made in the theoretical development with expected conditions for the seeker.

Specific tasks investigated and the results are identified below.

Task a) Develop theory for the purpose of treating the atmosphere between an imaging device and an object as an optical window to which techniques for quantitative description such as the modulation transfer function can be applied. Structure theory into a mathematical model defining key parameters which must be known.

Results of Task a)

Starting with Maxwell's equations (32)-(35) and proceeding through a series of assumptions and approximations we arrive at equation (73), the scalar Helmholtz wave equation, which describes optical propagation in the atmosphere. Several solution methods to this second-order differential equation are examined. A rigorous treatment based on Green's theorem is described in Section 3.2.4, yielding equation (130), which describes the optical amplitude function at $z = d$ in terms of the input optical amplitude at $z = 0$ and atmosphere dependent parameters.

Section 4.1 extends the result to give intensity in the image plane of a simple imaging system. From this is computed the atmosphere-lens point spread function (equation 157), the optical transfer function (equation 168) and the MTF (equation 169).

Task b) Develop theory for comparing atmospheric effects on the modulation transfer function (MTF) and on the reduction of contrast. Specifically examine the

effects of spatial frequency on the reduction of contrast by the atmosphere and relate the theories for MTF and contrast transfer function (CTF).

Results of Task b)

Contrast based on the model developed in task a) is computed which includes dependence on spatial frequency content of the target object, atmospheric parameters and ambient irradiance in Section 4.4. Universal contrast and modulation contrast through the atmosphere are given for a periodic bar chart in equations (190) and (192). Appendix C gives further details on MTF and CTF.

Task c) Develop a mathematical model for treating the effects of the atmosphere for the finite spectral band application (i.e. non-monochromatic). Emphasis will be on the visible and near infrared spectral regions (0.4 to 1.2 micrometers).

Results of Task c)

The theory of Duntley and Middleton for contrast reduction is extended to the finite spectral band case in Section 2.2 by defining a broadband extinction coefficient. Atmospheric transmittance is then compared for the new theory versus transmittance at a central wavelength. The error is small except for marginal conditions (i.e., slant range near the visibility range).

Task d) Develop, where needed, techniques for measuring the parameters required in an atmospheric MTF model. Recommend hardware which presently exists or hardware which must be developed for measuring the desired parameters.

Results of Task d)

Parameter measurement methods are discussed in Section 5.0 for atmospheric extinction coefficient and atmospheric refractive index structure function. Additionally, measurement methods for determining spatial size dependence on contrast transfer are discussed briefly. This effort did not get to the point of defining measurement hardware. Additional measurement program planning is needed.

Task e) Having developed and proven validity of an atmospheric MTF model in the visual region of the light spectrum, extend model to encompass the near infrared (0.7 to 0.9) micron region).

Results of Task e)

With respect to this task, no validation was accomplished. Further, the theory as developed is not restricted to the visible spectrum. All the assumptions should still be valid in the near infrared region (0.7 to 0.9 micron). The major effect of wavelength should be a change in certain parameter values.

Measurement methods for determining atmospheric imaging parameters are discussed for the simple contrast reduction theory of Duntley and Middleton and for the field theory development of resolution and contrast dependence on all target and atmospheric parameters.

Finally, some conclusions about the study are drawn and discussed in terms of areas needing further work.

Rigorous solution of the general effects of the atmosphere on propagating electromagnetic energy requires the solution of Maxwell's equations for a time-varying, inhomogeneous medium subject to a given set of boundary conditions. The atmosphere is assumed to be locally isotropic since measurement data tend to agree with this assumption. Even so, only approximate solutions are available and the validity of each approximation method must be checked against the parametric definition for a specific application. For example, early attempts to study nrcpagation in a random

medium used a geometric-optics approximation which was unfortunately valid for only very short optical paths. A later development by Tatarski [1] which is based on the Rytov approximation and which has been widely used provided a much greater range of validity. However, for propagation paths in excess of one kilometer, even the Rytov approximation begins to diverge from experimentally derived propagation measurements. Based on the work of Lee and Harp [2], which shows the Rytov approximation to be equivalent to the scatter of the incident wave by a series of random phase screens, the divergence from measured values at long optical paths is due to a neglect of multiple scattering effects of small regions of turbulence along the optical path.

Of the several methods developed for overcoming the limitations of the Rytov method, the most promising is the Markov approximation [3]-[5] which has validity over ranges in excess of hundreds of kilometers. For a detailed comparison of the above approximations see Fante [6].

Three separate but related phenomena contribute to the effects of the atmosphere on optical radiation: absorption, scattering and refractive index fluctuations. Absorption and scattering are typically related to the effects of atmospheric gas and particle constituents; whereas, refractive index fluctuations are typically related ^{to turbulence effects} (i.e. density gradients, temperature, and pressure differences). Absorption and scattering effects on contrast between an object and its background is based on work by Koschmieder [7] which was later simplified in a paper by Duntley [8] to give the "two-constant" theory. That is, the reduction of contrast by the atmosphere is adequately represented by two parameters for most "seeing"

conditions. The two parameters are not in fact constants but depend on optical wavelength and on the specific geometry of the observation scenario. Middleton [9] present an easy to read development of the Koschmieder theory and of Duntley's "two-constant" theory. In addition he presents arguments for eliminating certain restrictive assumptions made by Duntley. The work by Duntley and by Middleton still is the basis for contrast reduction formulas cited in more recent treatises on the optical effects of the atmosphere [10].

The second body of knowledge found in the literature treats the effects of atmospheric turbulence on MTF and typically makes no mention of what relation this has to contrast. Recent works [11]-[13], however, include absorption and ambient light in developing the EM theory approach to imaging in an inhomogeneous medium. The work of Duntley and Middleton in turn develops no relation of contrast reduction to resolution or MTF effects of the atmosphere. From their definitions we know that there is a close relationship between MTF and Contrast Transfer Function (CTF). So the question which arises is, "Do the two apparently different results derived from different approaches to the problem actually give the same information?" This question will be examined later in this report after a more detailed outline is given of results of the two different approaches.

The basic primer for work relating to the MTF of the atmosphere is the book by Taturski [1]. A later book by the same author [14] extends results presented in the first book and includes a discussion

of the Markov approximation method. More recent works have expanded, modified and given different approaches to the basic work of Tatarski. In most cases MTF of the atmosphere is examined by first analyzing the point spread function of a lens for which the input wave exhibits a random log-amplitude and phase variation across the lens aperture. The MTF is then the Fourier transform of the point spread function. Since the log amplitude and phase are random functions, only statistically averaged results are obtainable. Additionally for short exposure times the results are such that the effect due to the atmosphere is not separable from the effect due to the lens. Fried [15] has presented a development which starts with fundamental relations given in Born and Wolf [16] and proceeds to final MTF using concepts and techniques which are more familiar to electrical engineers. He develops expressions for both short-time and long-time exposure. Hufnagel and Stanley [17] develop an optical transfer function for the atmosphere based on computation of the spatial mutual coherence function for an atmospherically refracted wavefront. Their result is based on a long exposure time so that the MTF is accurately represented by an ensemble average with respect to the log-amplitude and phase random functions. Fried's work [15] which was done later agrees with the Hufnagel and Stanley result for a long-time exposure case. A recent article by Fante [6] serves as a good summary review of the optical effects of atmospheric turbulence and offers a rather extensive bibliography.

An extension of the monochromatic theory for contrast reduction as developed by Duntley [8] and Middleton [9] was accomplished and results

are described in this report. Example calculations show that the error incurred by using a monochromatic representation for the broadband contrast reduction of the atmosphere is very small for typical imaging scenarios. This result is in agreement with the opinions of other investigators [18] that wavelength effects are minor effects over a relatively narrow spectral region (as, for example, the spectral region of sensitivity for an imaging seeker). A recent article [19] describes the effects of the atmosphere on optical signals which exhibit partial spatial coherence (such as might be observed by illuminating a rough surface with a laser); however, the analysis is still for monochromatic radiation.

Also, in this report, a detailed theoretical development based on Maxwell's equations is given which describes the optical properties of the atmosphere. The development includes absorption. From the resultant atmospheric Green's function, the propagation of complex spatial optical amplitude functions from the object to the image plane of a simple lens is given. The atmosphere-lens MTF is then computed from the spatial Fourier transform of the optical intensity function in the image plane.

Contrast in the image plane is computed in terms of the definition for universal contrast using the image plane intensity and adding a term for ambient irradiance. The theory as developed gives a rigorous treatment of the contrast transfer and MTF of an atmosphere-lens imaging system for the monochromatic case. In terms of extending the theory to finite spectral band imaging the following apply. The development is general up to a point at which an assumption of monochromaticity was made. Thus the theoretical development to that point applies to the broadband case. It remains to extend the model to give contrast and MTF for the

broadband case.

Measurement methods for determining atmospheric imaging parameters are discussed in Section 5.0 for the simple contrast reduction theory of Duntley and Middleton and for the field theory development of resolution and contrast dependence on all target and atmospheric parameters.

Finally, some conclusions about the study are drawn and discussed in terms of areas needing further work.

II. CONTRAST REDUCTION BASED ON A THERMODYNAMIC APPROACH

The development for optical contrast reduction by the atmosphere is based on work by Duntley [8] and Middleton [9] and assumes the scattering of radiation into and out of a given optical path can be adequately represented by two constants. These two constants are the atmospheric extinction coefficient, σ , and the ambient air light, L_a . The extinction coefficient includes scattering and absorption and is dependent on wavelength as well as the composition and size of atmospheric constituents. Thus σ will depend on many separate mechanisms which in general are difficult to predict theoretically. With the exception of the relatively restrictive cases of Rayleigh and Mie scattering most data on extinction coefficient values is based on empirical measurements. The meteorological range, V , is defined as the range at which the contrast reduction due to the atmosphere is 0.02. Also called the "visibility range" or just "visibility", V has units of distance and has been assigned values for several subjective descriptions of "seeing conditions". For example a standard clear day is one for which the visibility is 23.5 km.

A well-presented and easily readable review of the physics of light scattering is given by Harris [20]. His definition for scattering is "a process by which radiant energy incident on an arbitrary medium is redistributed into one or more angular directions of propagation". He includes absorption as a scattering mechanism in addition to refraction, reflection and diffraction. Incident light will be refracted and/or

reflected at a refractive index boundary. The angles and their relationship to the refractive indices are given by the Fresnel formulae found in most elementary optical references. Diffraction is the bending of light rays which pass close to an edge or opening. For the case of the atmosphere, light will be diffracted when passing close to particles in the air. Finally a number of absorption mechanisms may contribute to the overall scattering process. Resonance absorption and re-emission changes only the direction of the radiation; wavelength is unchanged. Fluorescence and Raman absorption on the other hand result in a change in wavelength for the re-radiated photon. Total absorption of a photon by a particle results in an increase of the internal energy of the particle which essentially re-distributes the radiated energy over a blackbody-like spectrum.

The rigorous treatment of scattering is again based on solutions to Maxwell's equations. The Mie theory develops solutions for the case of small spherical particles. Rayleigh theory develops solutions for very small particles (actually the Rayleigh theory applies primarily to molecules of the atmospheric gases and not to particles, and shows an inverse sixth power dependence on spherical radius for particles ≤ 0.1 micron for visible radiation.) The details of a scattering situation are a complex function of particle size, particle shape, the complex index of refraction and angle.

Measurement of scattering cross-sections from few particles of a given size is extrapolated to more dense particle populations directly since laboratory and field measurements indicate the validity of this

of this approach. In other words, the spacing between particles is much greater than the particle size and the optical wavelength so that there is no interaction between particles. However, for scattering in high particle concentrations such as a dense cloud this is not quite true and the total scattering effect will be smoothed by multiple scattering. In fact this effect is more pronounced at optical wavelengths than at infrared wavelengths so that seeing conditions in a very hazy atmosphere will be better at infrared wavelengths. The degree of haziness at which this advantage for the infrared becomes obvious is not well defined and is strongly dependent on the specific atmospheric conditions.

A. The Theory of Duntley and Middleton

The following development of the contrast reduction effects of the atmosphere is based on the original work of Duntley as later modified by Middleton. The purpose here is to give a brief theoretical development, identify the general result and discuss special cases, in particular the sky-to-ground observer.

The model for contrast reduction due to the atmosphere for a sky-to-ground observer is shown in Figure 1. For a differential lamina of air perpendicular to the path of sight we have

$$L(r) + \frac{dL(r)}{dr} dr = L(r + dr) \quad (1)$$

where: $L(r + dr)$ is the radiance out of the lamina toward the observer and $dL(r)/dr$ is the change in path radiance in the lamina.

Clearly, radiance can be lost or gained in the lamina dr , even in the absence of sources within dr which is the case assumed for the atmosphere.

Then $dL(r)/dr$ depends on two things: 1) radiance lost through extinction (scattering and absorption) and 2) radiance gained as air light from the surrounding medium which has been scattered into the path of sight.

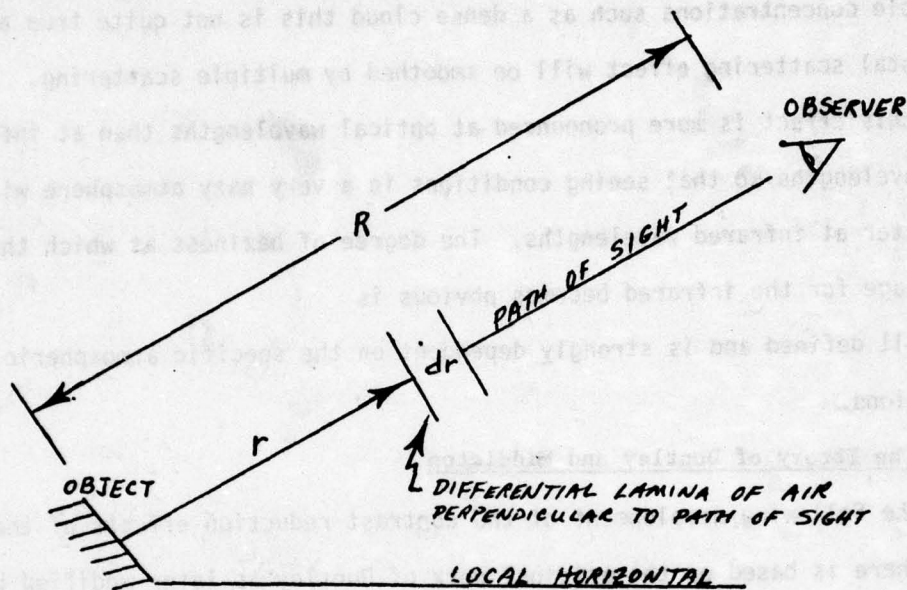


Figure 1. Model for contrast reduction.

Then

$$\frac{dL(r)}{dr} = -\sigma(r)L(r) + L_a(r) \quad (2)$$

where: σ = extinction coefficient

$$\sigma = \alpha + \beta$$

α = absorption coefficient

β = scattering coefficient

$L_a(r)$ is the air light scattered into the path of sight

In arriving at the differential equation in (2) a number of assumptions are inherent. First it is assumed that the light scattered by a lamina is

directly proportional to the incident light. This assumption is implied by the argument that there is no interaction among molecules or particles in terms of scattering. Certainly for a reasonable range of particle densities and incident radiation levels this superposition theory is expected to hold true. Other assumptions in the development of (2) are that the object be small in comparison to the range R and that second order scattering of light from the object into the path of sight is inseparable from and a part of the term for air light, $L_a(r)$. Additionally it should be emphasized that the terms in (2) are dependent not only on r as indicated but on optical wavelength as well. Without knowing the specific functional form of the " r "-dependence we can go no further. Both Duntley and Middleton provide a convenient way for avoiding this difficulty. It is based on the assumption that $\sigma(r)$ and $L_a(r)$ both have the same functional dependence on r so that

$$\begin{aligned}\sigma(r) &= \sigma_0 f(r) \\ L_a(r) &= L_a(0)f(r)\end{aligned}\tag{3}$$

where $f(r)$ is some unspecified function of r .

σ_0 and $L_a(0)$ are constants and represent their respective values at $r = 0$. With this assumption (2) becomes

$$\int_{L_0}^{L_R} \frac{dL(r)}{\sigma_0 L(r) - L_a(0)} = - \int_0^R f(r) dr\tag{4}$$

where the right hand side of (4) is designated \bar{R} and is called the "optical slant range". Then the solution to (4) becomes

$$L_R = \frac{L_a(0)}{\sigma_0} \left[1 - e^{-\sigma_0 \bar{R}} \right] + L_0 e^{-\sigma_0 \bar{R}}\tag{5}$$

RADIANCE EFFECTS OF THE ATMOSPHERE

L_0 is the inherent object radiance

L_R is the apparent object radiance at slant range R . Note that L_R has two terms. The first term represents an increase in apparent radiance due to air light being scattered into the optical path and the second term shows how the inherent object radiance is reduced by atmospheric extinction. Equation (5) is the basic result of Middleton and Duntley. Other expressions which relate contrast of the object with a background and for various viewing geometries occur more often in the literature than (5).

For example if we consider a localized background adjacent to the object (remember that the initial assumptions required the object, or background, to be small compared to R) then its apparent radiance at R is given by:

$$L_R' = \frac{L_a(0)}{\sigma_0} \left[1 - e^{-\sigma_0 \bar{R}} \right] + L_0' e^{-\sigma_0 \bar{R}} \quad (6)$$

And if we define contrast* as

$$C \triangleq \frac{L - L'}{L} \quad (7)$$

then the inherent contrast is

$$C_0 = \frac{L_0 - L_0'}{L_0'} \quad (8)$$

and the apparent contrast at R is

$$C_R = \frac{L_R - L_R'}{L_R'} \quad (9)$$

*There are several variations on the definition for contrast. See Appendix B.

Substituting (5) and (6) into (9) gives

$$C_R = \frac{(L_0 - L_0') e^{-\sigma_0 \bar{R}}}{L_R'} \quad (10)$$

and from (8) $L_0 - L_0' = C_0 L_0'$ so that

$$C_R = \frac{L_0'}{L_R'} C_0 e^{-\sigma_0 \bar{R}} \quad (11)$$

CONTRAST TRANSFER OF THE ATMOSPHERE

Equation (11) is the general result for contrast reduction by the atmosphere given by Middleton and by Duntley. This result was developed for the air-to-ground observer case; however, it is valid for the ground-to-air case except that the air-light contribution in equation (6) should be different for the two viewing directions.

For certain special cases where the background radiance is independent of range (the commonly cited example is for horizontal viewing against a sky background) we have that $L_0' = L_R'$ and (11) is written as

$$\frac{C_R}{C_0} = e^{-\sigma_0 \bar{R}} \quad (12)$$

SPECIAL CASE: RANGE INDEPENDENT BACKGROUND RADIANCE

An alternate expression for the air-to-ground case is given by substituting (6) into (11) for L_R' to give

$$\frac{C_R}{C_0} = \left[1 - \frac{L_a(0)}{L_0' \sigma_0} \left(1 - e^{\sigma_0 \bar{R}} \right) \right]^{-1} \quad (13)$$

which is further modified by recognizing $\frac{L_a(0)}{L_0' \sigma_0} = \frac{0.2}{\beta'}$ where β' is the

reflectivity of the background (see Middleton [9] p-71 for a discussion).

Thus

$$\frac{C_R}{C_0} = \left[1 - \frac{0.2}{\beta^T} \left(1 - e^{-\sigma_0 \bar{R}} \right) \right]^{-1} \quad (14)$$

is a valid expression which can also be shown to be equal to

$$\frac{C_R}{C_0} = \left[1 - \frac{0.2}{\beta^T} \left(\frac{1-T}{T} \right) \right]^{-1} \quad (15)$$

where $T = e^{-\sigma_0 \bar{R}}$ is the atmospheric transmittance over optical path \bar{R} .

B. Extension to a Finite Spectral Band

In evaluating the usefulness of theoretical expressions for contrast reduction by the atmosphere in terms of applicability for determining performance of imaging seekers or other broadband optical systems, it becomes necessary to examine the effect of spectral dependence. Some method for reconciling a monochromatic theory with a wideband, spectrally non-trivial application such as is true for visible and near-infrared imaging seekers must be found.

From the work of Duntley and Middleton, the spectral radiance transfer characteristic of the atmosphere is given for monochromatic radiation by

$$L_{\lambda}(R) = \frac{L_{a,\lambda}(0)}{\sigma_{\lambda}(0)} [1 - T_{\lambda}] + L_{\lambda}(0)T_{\lambda} \quad (16)$$

where $L_{\lambda}(R)$ is the spectral radiance at slant range R due to a source radiance $L_{\lambda}(0)$ at zero, $\sigma_{\lambda}(0)$ is the atmospheric extinction coefficient

at zero (including scattering and absorption) for a given wavelength, $L_{a,\lambda}(0)$ is the spectral radiance of the air light and T_λ is the atmospheric spectral transmittance given by

$$T_\lambda = \exp[-\sigma_\lambda(0)\bar{R}_\lambda] \quad (17)$$

where \bar{R}_λ is the optical path length from zero to R for wavelength λ . In (16) and (17) the subscript λ has been added to emphasize wavelength dependence for the various parameters.

For a finite band of wavelengths the total radiance $L_{\Delta\lambda}(R)$ is the summation over $\Delta\lambda$ of equation (16).

$$L_{\Delta\lambda}(R) = \int_{\Delta\lambda} \left\{ \frac{L_{a,\lambda}(0)}{\sigma_\lambda(0)} [1 - T_\lambda] + L_\lambda(0)T_\lambda \right\} d\lambda \quad (18)$$

Similarly, for the background radiance

$$L'_{\Delta\lambda}(R) = \int_{\Delta\lambda} \left\{ \frac{L_{a,\lambda}(0)}{\sigma_\lambda(0)} [1 - T_\lambda] + L'_\lambda(0)T_\lambda \right\} d\lambda \quad (19)$$

Then for contrast defined as $(L-L')/L'$ we get the broadband contrast at range R

$$C_{\Delta\lambda}(R) = \frac{\int_{\Delta\lambda} \{ [L_\lambda(0) - L'_\lambda(0)] T_\lambda \} d\lambda}{L'_{\Delta\lambda}(R)} \quad (20)$$

Further we have the broadband inherent contrast at range zero

$$C_{\Delta\lambda}(0) = \frac{\int_{\Delta\lambda} \{ L_\lambda(0) - L'_\lambda(0) \} d\lambda}{L'_{\Delta\lambda}(0)} \quad (21)$$

where $L'_{\Delta\lambda}(0) = \int_{\Delta\lambda} L'_\lambda(0) d\lambda$. From (20) and (21) we get

$$\frac{C_{\Delta\lambda}(R)}{C_{\Delta\lambda}(0)} = \left[\frac{L'_{\Delta\lambda}(0)}{L'_{\Delta\lambda}(R)} \right] T_{\Delta\lambda} \quad (22)$$

Equation (22) is in the same form as the general result derived by Duntley for the monochromatic case, where $T_{\Delta\lambda}$ is now the mean broadband atmospheric transmittance. We can equate $T_{\Delta\lambda}$ with $\exp[-\sigma_{\Delta\lambda}(0)\bar{R}_{\Delta\lambda}]$ where $\sigma_{\Delta\lambda}(0)$ can be called the mean broadband extinction coefficient of the optical path. However, $\sigma_{\Delta\lambda}(0)$ is not a simple average value within the spectral band of interest but is given by

$$\sigma_{\Delta\lambda}(0) = \frac{-1}{\bar{R}_{\Delta\lambda}} \ln \left[\frac{\int_{\Delta\lambda} \{L_\lambda(0) - L'_\lambda(0)\} T_\lambda d\lambda}{\int_{\Delta\lambda} \{L_\lambda(0) - L'_\lambda(0)\} d\lambda} \right] \quad (23)$$

The result in (23) is easily evaluated by computer if all the spectral data are available. For the special case of flat spectral emission by the object and background we get the simpler result which is independent of source and background radiance levels

$$\sigma_{\Delta\lambda}(0) = \frac{-1}{\bar{R}_{\Delta\lambda}} \ln \left\{ \frac{1}{\Delta\lambda} \int_{\Delta\lambda} \exp[-\sigma_\lambda(0)\bar{R}_\lambda] d\lambda \right\} \quad (24)$$

Further, if we look at the derivation for \bar{R} in Duntley's paper [8] we have the following

$$\sigma_\lambda(r) = \sigma_\lambda(0)f(r)$$

and

$$\bar{R} = \int_0^R f(r) dr$$

(25)

where the unspecified function $f(r)$ was assumed to give range dependence for both the extinction coefficient and the air light, $L_{a,\lambda}(r)$. It seems reasonable to argue that the spectral dependence for $\sigma_\lambda(r)$ is contained in the $\sigma_\lambda(0)$ term so that $f(r)$ and, hence \bar{R} , are not functions of wavelength. With this argument the λ subscript can be dropped from \bar{R} so that

$$\bar{R}_{\Delta\lambda} = \bar{R}_\lambda = \bar{R} \quad (26)$$

Then (23) becomes

$$\sigma_{\Delta\lambda}(0) = \frac{1}{\bar{R}} \ln \left[\frac{\int_{\Delta\lambda} \{L_\lambda(0) - L'_\lambda(0)\} d\lambda}{\int_{\Delta\lambda} \{[L_\lambda(0) - L'_\lambda(0)] T_\lambda\} d\lambda} \right] \quad (27)$$

or for the case of flat target and background spectral radiance (24) is given by

$$\sigma_{\Delta\lambda}(0) = \frac{-1}{\bar{R}} \ln \left\{ \frac{1}{\Delta\lambda} \int_{\Delta\lambda} \exp[-\sigma_\lambda(0)\bar{R}] d\lambda \right\} \quad (28)$$

The result in (28) is compared with the value of extinction coefficient obtained as a simple average value across $\Delta\lambda$ by the ratio

$$E = \frac{\sigma_{\Delta\lambda}(0)}{\sigma_{\text{AVG}}(0)} = \frac{-\frac{1}{\bar{R}} \ln \left\{ \frac{1}{\Delta\lambda} \int_{\Delta\lambda} \exp[-\sigma_\lambda(0)\bar{R}] d\lambda \right\}}{\frac{1}{\Delta\lambda} \int_{\Delta\lambda} \sigma_\lambda(0) d\lambda} \quad (29)$$

The comparison is made by choosing a model for $\sigma_\lambda(0)$ in the visible spectrum. Several authors (for example [21] and [22] have suggested models of the form

$$\sigma_\lambda(0) = \sigma_{0.55}(0) \left(\frac{a}{\lambda} \right)^q \quad (30)$$

where $\sigma_{0.55}(0)$ is the extinction coefficient at a wavelength of 0.55 micrometers for a given visibility, "a" is 0.55 or 0.61 micrometers and "q" is a constant in the range 0.7 to about 3.0 for typical visibility values.

The effect on atmospheric transmittance of a broadband theory is further compared with monochromatic theory using average values for $\sigma_{\lambda}(0)$ by the ratio

$$\rho = \frac{T_{\Delta\lambda}}{T_{AVG}} = \frac{\exp[-\sigma_{\Delta\lambda}(0)\bar{R}]}{\exp[-\sigma_{AVG}(0)\bar{R}]} \quad (31)$$

Values for ρ are shown in Figure 2 for various visibilities as a function of slant range. Parameter values $a = 0.55$ and $q = 0.7$ were used in the computation.

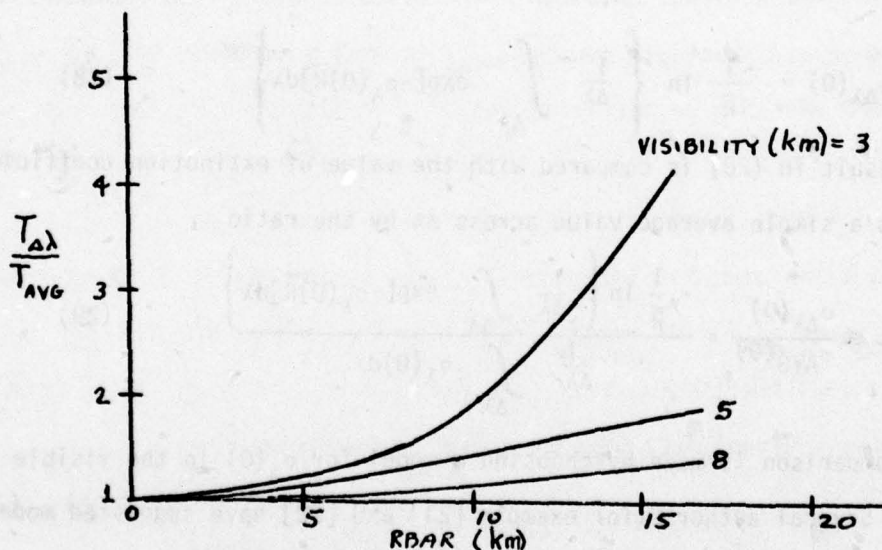


Figure 2. Ratio of broadband to average transmittance for various optical slant ranges and visibilities.

The result in (22) is dependent on an equivalent broadband extinction coefficient $\sigma_{\Delta\lambda}(0)$, which is dependent on the spectral radiance levels of the target and background as well as the spectral extinction coefficient $\sigma_{\lambda}(0)$. Rigorous computation of the broadband contrast reduction is easily achieved by computer if all the spectral data are available.

A special case where target and background radiances are spectrally flat yields the results in Figure 2. For high visibility, little error is encountered in the atmospheric transmittance by using an average value for the extinction coefficient. For poor visibility at longer ranges the effect becomes more pronounced. It is emphasized that these results are for simplified (spectrally smooth) models for target/background radiances and spectral extinction coefficients. Models which included distinct spectral characteristics should have a greater effect on the approximation error, ρ .

A major assumption made in the development is one made by Duntley and by Middleton in assuming that both the extinction coefficient $\sigma_{\lambda}(R)$ and the ambient light scattered into the optical path $L_{\alpha,\lambda}(R)$ have the same functional dependence on range R . Extinction coefficient includes effects of both scattering and absorption and depends on wavelength, particle size (r') and refractive index (m). Additionally the scattered and absorbed radiation exhibits an angular distribution which is dependent on these same parameters. Examples of the dependence of scattering and absorption on the size parameter $a = 2\pi r'/\lambda$ for various values of m and scattering angles are given in the excellent summary paper by Harris [20].

The "air light" term, $L_{a,\lambda}(R)$, represents radiation scattered into the line of sight by particles along the path. This term is then dependent primarily on the scattering properties along the path plus the angular distribution of ambient radiation. It is easy to visualize conditions for which $\sigma(R)$ and $L_{a,\lambda}(R)$ have significantly different range dependencies.

However, the major thrust of this development was to demonstrate that for certain imaging applications where sensitivity over the visible spectrum exists, some mechanism for including spectral dependence in contrast reduction calculations is desirable.

In summary, the following points are made with respect to Figure 2.

a) A value of $T_{\Delta\lambda}/T_{AVG}$ greater than unity shows that the atmospheric transmittance is larger using the broadband model than if an average extinction coefficient is used. This improves contrast slightly.

b) The method for finding T_{AVG} is more rigorous than some averaging techniques, eg., one method is to use the value of σ at the central wavelength in the spectral passband.

c) The dependence of σ on wavelength was modeled as a smooth function. In some spectral regions, σ exhibits more distinct wavelength dependence.

d) For the case examined, Figure 2 shows that T_{AVG} and $T_{\Delta\lambda}$ are essentially equal for slant ranges less than the visibility range.

e) The results given are for an assumed flat spectral radiance for both the target and background.

f) Any changes in b.), c.) and e.) above will affect the result shown in Figure 2. The exact effect is strongly case dependent.

III. OPTICS OF THE ATMOSPHERE BASED ON ELECTROMAGNETIC FIELD THEORY

In this section, beginning with Maxwell's equations, a detailed development of the theory behind effects of the atmosphere on optical signals is given.

First, in 3.1, the vector Helmholtz wave equation describing vector electric and magnetic fields for the optical signal is derived from Maxwell's equations, several vector identities, assumptions as to the nature of the optical path and from Fourier theory for a monochromatic optical waveform. Next the scalar monochromatic theory is derived from the vector theory which gives the scalar Helmholtz wave equation. It describes the spatial behavior of an optical complex amplitude propagating in an isotropic, linear, slightly inhomogeneous, slightly absorbing medium (i.e. the atmosphere). Solutions to the scalar Helmholtz equation subject to a set of boundary conditions are then approximated. A number of methods have been used to approximate a solution to the Helmholtz equation. Several of these are outlined briefly. A conceptually satisfying solution is found using a Green's function approach. The solution is the Rayleigh-Sommerfeld integral extended to include inhomogeneities and absorption. A further assumption produces the modified form of the Rayleigh-Sommerfeld integral or the extended Huygen-Fresnel Integral. This method is given in considerable detail.

A. Maxwell's Equations and the Inhomogeneous Scalar Helmholtz Equation

Using rationalized MKS units, we start with Maxwell's equations [23]

$$\nabla \times \vec{H} = \dot{\vec{D}} + \vec{J} \quad (32)$$

$$\nabla \times \vec{E} = -\dot{\vec{B}} \quad (33)$$

$$\nabla \cdot \vec{D} = \rho \quad (34)$$

$$\nabla \cdot \vec{B} = 0 \quad (35)$$

Isotropic, Linear Medium

$$\vec{D} = \epsilon(x, y, z)\vec{E} \quad (36)$$

$$\vec{B} = \mu(x, y, z)\vec{H} \quad (37)$$

$$\vec{J} = \sigma(x, y, z)\vec{E} \quad (38)$$

divide (33) by μ and use (37)

$$\frac{1}{\mu} \nabla \times \vec{E} = -\dot{\vec{H}} \quad (39)$$

Take curl of (39)

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \vec{E} \right) = -\nabla \times \dot{\vec{H}} \quad (40)$$

differentiate (32) wrt time

$$\nabla \times \dot{\vec{H}} = \ddot{\vec{D}} + \dot{\vec{J}} \quad (41)$$

use (36) and (38) in (41) and if ϵ and σ not fcns. of time[†]

$$\nabla \times \dot{\vec{H}} = \epsilon \ddot{\vec{E}} + \sigma \dot{\vec{E}} \quad (42)$$

where x, y, z dependence of ϵ, σ and μ will be left implicit for convenience.

[†] In general, ϵ and σ , as well as the refractive index to be defined from them, will depend on x, y, z and on "t". However, this time dependence is usually assumed to be associated with atmospheric winds and according to "Taylor's frozen flow hypothesis" is statistically dependent on $[r - V(r)t]$ where $r = (x, y, z)$ and $V(r)$ is the local wind velocity. For the development here, we take the approach of most authors and ignore the time dependence in obtaining approximate solutions to the wave equation. Time dependence is considered later in the development in terms of specific exposure times and expected effects on MTF.

Substitute (42) into (40)

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \bar{E} \right) = -(\epsilon \ddot{\bar{E}} + \sigma \dot{\bar{E}}) \quad (43)$$

identity

$$\nabla \times (u\bar{V}) = u\nabla \times \bar{V} + \nabla u \times \bar{V} \quad (44)$$

u is a scalar

\bar{V} is a vector

Using (44) in (43) where $u = \frac{1}{\mu}$ and $\bar{V} = \nabla \times \bar{E}$

$$\frac{1}{\mu} \nabla \times \nabla \times \bar{E} + \nabla \left(\frac{1}{\mu} \right) \times \nabla \times \bar{E} = -(\epsilon \ddot{\bar{E}} + \sigma \dot{\bar{E}}) \quad (45)$$

identity

$$\nabla \times \nabla \times \bar{V} = \nabla (\nabla \cdot \bar{V}) - \nabla^2 \bar{V} \quad (46)$$

using (46) in (45) gives

$$\frac{1}{\mu} \nabla (\nabla \cdot \bar{E}) - \frac{1}{\mu} \nabla^2 \bar{E} + \nabla \left(\frac{1}{\mu} \right) \times \nabla \times \bar{E} = -(\epsilon \ddot{\bar{E}} + \sigma \dot{\bar{E}}) \quad (47)$$

From (34), (36) and using the

$$\text{identity} \quad \text{div } u\bar{V} = \nabla \cdot (u\bar{V}) = u\nabla \cdot \bar{V} + \bar{V} \cdot \nabla u \quad (48)$$

$$\nabla \cdot \bar{D} = \nabla \cdot (\epsilon \bar{E}) = \epsilon \nabla \cdot \bar{E} + \bar{E} \cdot \nabla \epsilon = \rho \quad (49)$$

Solving (49) for $\nabla \cdot \bar{E}$

$$\nabla \cdot \bar{E} = \frac{1}{\epsilon} [\rho - \bar{E} \cdot \nabla \epsilon] \quad (50)$$

Substitute (50) into (47)

$$\frac{1}{\mu} \nabla \left(\frac{1}{\epsilon} (\rho - \bar{E} \cdot \nabla \epsilon) \right) - \frac{1}{\mu} \nabla^2 \bar{E} + \nabla \left(\frac{1}{\mu} \right) \times \nabla \times \bar{E} = -(\epsilon \ddot{\bar{E}} + \sigma \dot{\bar{E}}) \quad (51)$$

Multiply (51) by μ

$$\nabla \left(\frac{1}{\epsilon} [\rho - \bar{E} \cdot \nabla \epsilon] \right) - \nabla^2 \bar{E} + \mu \nabla \left(\frac{1}{\mu} \right) \times \nabla \times \bar{E} + \mu [\epsilon \ddot{\bar{E}} + \sigma \dot{\bar{E}}] = 0 \quad (52)$$

or rearranging and changing sign

$$\nabla^2 \bar{E} - \mu \epsilon \ddot{\bar{E}} - \mu \sigma \dot{\bar{E}} - \mu \nabla \left(\frac{1}{\mu} \right) \times \nabla \times \bar{E} + \nabla \left[\frac{1}{\epsilon} \bar{E} \cdot \nabla \epsilon \right] - \nabla \left[\frac{\rho}{\epsilon} \right] = 0 \quad (53)$$

Result in (53) is correct for a linear isotropic medium (i.e. all terms in \vec{E} are of power one, ρ & μ & ϵ & σ may depend on x , y , & z but will be same at a given point for all components of the vector \vec{E}).

An alternate form of (53) is achieved by recognizing that

$$\nabla\left(\frac{1}{\mu}\right) = -\frac{1}{\mu^2} \nabla\mu \quad (54)$$

$$\nabla(\log \mu) = \frac{\partial(\log \mu)}{\partial \mu} \nabla\mu = \frac{1}{\mu} \nabla\mu \quad (55)$$

From (54) and (55) we see the term in (53)

$$[-\mu \nabla \left(\frac{1}{\mu}\right)] \text{ becomes } -\mu \left(-\frac{1}{\mu^2} \nabla\mu\right) = \frac{1}{\mu} \nabla\mu = \nabla(\log \mu) \quad (56)$$

Further since

$$\vec{A} \cdot \frac{\nabla u}{u} = \frac{1}{u} \vec{A} \cdot \nabla u \quad (57)$$

and $\nabla(\log \epsilon) = \frac{1}{\epsilon} \nabla\epsilon \quad (58)$

the term $\frac{1}{\epsilon} \vec{E} \cdot \nabla\epsilon$ becomes $\vec{E} \cdot \nabla(\log \epsilon) \quad (59)$

Using (56) and (59) in (53) gives

Vector D.E. for electric field \vec{E} , Linear, Isotropic Medium

$$\nabla^2 \vec{E} - \mu \epsilon \ddot{\vec{E}} - \mu \sigma \dot{\vec{E}} + \nabla(\log \mu) \times \nabla \times \vec{E} + \nabla[\vec{E} \cdot \nabla(\log \epsilon)] - \nabla\left(\frac{\rho}{\epsilon}\right) = 0$$

SPECIAL CASE - VACUUM

(60)

$$\mu = \text{constant}, \epsilon = \text{constant} \rightarrow \nabla(\log \mu) = \nabla(\log \epsilon) = 0$$

$$\sigma = 0, \rho = 0$$

zero conductivity source-free

Result is Vector Helmholtz equation

$$\nabla^2 \vec{E} - \mu \epsilon \ddot{\vec{E}} = 0 \quad (61)$$

SPECIAL CASE - ATMOSPHERE

Conditions (1) Weakly inhomogeneous - i.e. μ , ϵ , & σ show slight variations with x , y & z ; however, the variation path length is large compared to spatial variation in \vec{E} (\sim wavelength)

Result of (1): in (60) $\nabla(\log \mu)$ & $\nabla(\log \epsilon)$ are small and approximated as zero.

(2) Weakly absorbing (conducting) - i.e. charge density ρ induced by absorption of energy is small and varies slowly with x , y , z .

Result of (2): $\nabla(\frac{\rho}{\epsilon})$ is small and approximated as zero.

Then eqn (60) becomes

$\nabla^2 \vec{E} - \mu \epsilon \ddot{\vec{E}} - \mu \sigma \dot{\vec{E}} = 0$	(62)
Weakly in-homogeneous, weakly absorbing medium (linear and isotropic also)	

To handle time derivatives, take temporal Fourier transform of $\vec{E}(x,y,z,t)$

$\Leftrightarrow \hat{\vec{E}}(x,y,z,\nu)$, ν is optical frequency (Hz)

i.e. the defining equations for temporal transform are

Temporal Fourier Transform Pair

$$\hat{\vec{E}}(x,y,z,\nu) \stackrel{\Delta}{=} \int_{-\infty}^{\infty} \vec{E}(x,y,z,t) e^{j2\pi\nu t} dt \quad (63)$$

$$\vec{E}(x,y,z,t) \stackrel{\Delta}{=} \int_{-\infty}^{\infty} \hat{\vec{E}}(x,y,z,\nu) e^{-j2\pi\nu t} d\nu \quad (64)$$

If (64) is differentiated wrt time once & twice, the result is

$$\dot{\vec{E}} = \int_{-\infty}^{\infty} [-j2\pi\nu \hat{\vec{E}}(x,y,z,\nu)] e^{-j2\pi\nu t} d\nu \quad (65)$$

$$\ddot{\vec{E}} = \int_{-\infty}^{\infty} [(-j2\pi\nu)^2 \hat{\vec{E}}(x,y,z,\nu)] e^{-j2\pi\nu t} d\nu \quad (66)$$

Thus we have the following transform pairs

$$\begin{aligned}\hat{E}(x,y,z,v) &\Leftrightarrow \bar{E}(x,y,z,t) \\ -j2\pi v \hat{E}(x,y,z,v) &\Leftrightarrow \dot{\bar{E}}(x,y,z,t) \\ (-j2\pi v)^2 \hat{E}(x,y,z,v) &\Leftrightarrow \ddot{\bar{E}}(x,y,z,t)\end{aligned}\quad (67)$$

Using (67) and taking the Fourier transform of (62) gives

$$\nabla^2 \hat{E} + \mu \epsilon (2\pi v)^2 \hat{E} + \mu \sigma (j2\pi v) \hat{E} = 0$$

$$\text{or} \quad [\nabla^2 + (2\pi v)^2 \mu (\epsilon + j \frac{\sigma}{2\pi v})] \hat{E} = 0 \quad (68)$$

$\hat{E} = \hat{E}(x,y,z,v)$

-Vector Helmholtz Wave Equation- Weakly inhomogeneous Weakly Absorbing Isotropic Linear Medium
--

The term in parenthesis

$$(\epsilon + j \frac{\sigma}{2\pi v}) = \hat{\epsilon} \quad (69)$$

can be thought of as a complex permittivity in which case (68) is identical to the temporal Fourier transform of the vacuum equation (61).

NOTE: In general for a non-dispersive medium ϵ does not depend on optical frequency v ; however, as seen by (69) the complex permittivity $\hat{\epsilon}$ does depend on v . This is another way of saying that optical absorption (represented by the imaginary part of $\hat{\epsilon}$) is wavelength dependent which has been verified by experiment.

With (69), (68) becomes

$$[\nabla^2 + (2\pi v)^2 \mu \hat{\epsilon}] \hat{E} = 0 \quad (70)$$

Vector Helmholtz eqn for practical atmosphere

Solutions are sought to (70)

First it is recognized that (70) describes only the electric field behavior of an electromagnetic wave. It can be shown by a development similar to that above that the magnetic field vector \vec{H} is described by a similar second order D.E.

Second, note is made that both \vec{H} and \vec{E} can have three scalar components, eg. in rectangular coordinates

$$\left. \begin{array}{ll} E_x(x,y,z,v) & H_x(x,y,z,v) \\ E_y(x,y,z,v) & H_y(x,y,z,v) \\ E_z(x,y,z,v) & H_z(x,y,z,v) \end{array} \right\} \quad (71)$$

must all be solved simultaneously subject to a set of boundary conditions. Each component in (71) is described by an equation of the form of (70). Finally, simultaneous solution of the components in (71) has been achieved for a very few special cases of little practical interest. For cases of practical interest we develop the following scalar theory.

Postulate a scalar field $\phi(x,y,z,t)$ and its temporal Fourier transform $\hat{\phi}(x,y,z,v)$ such that ϕ "represents" a real optical field. ϕ may be thought of as the scalar amplitude of a vector field such as

$$\phi = \sqrt{E_x^2 + E_y^2 + E_z^2} \quad (72)$$

Thus letting ϕ represent the optical field we find that $\hat{\phi}$ satisfies equation (70)

	$[\nabla^2 + (2\pi v)^2 \mu \epsilon] \hat{\phi}(x,y,z,v) = 0$	(73)
Practical Atmo- sphere	Scalar Helmholtz wave equation, weakly inhomogeneous, weakly absorbing, linear, isotropic medium	

Definition of some terms

$$k \triangleq \frac{1}{\lambda} = \text{wavenumber (m}^{-1}\text{)}$$

$$c = \text{speed of light (m/sec)}$$

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \text{velocity (m/sec)}$$

$$\hat{v} = 1/\sqrt{\mu\hat{\epsilon}}$$

$$\nu = \text{frequency (sec}^{-1}\text{)} = \frac{c}{\lambda} = kc$$

$$n = \text{refractive index} \triangleq \frac{c}{v} = c\sqrt{\mu\epsilon}$$

$$\hat{n} = \text{complex refractive index} = \frac{c}{\hat{v}} = c\sqrt{\mu\hat{\epsilon}}$$

$$\text{Then } (2\pi\nu)^2 \mu\hat{\epsilon} = (2\pi)^2 k^2 \hat{n}^2 \quad (74)$$

Using (74) in (73) gives

$$\boxed{[\nabla^2 + (2\pi)^2 k^2 \hat{n}^2] \hat{\phi}(x, y, z, \nu) = 0} \quad (75)$$

Alternate form of (73)

B. Solution Methods for the Helmholtz Equation

Solutions are sought to equation (39). Consider first the monochromatic case, i.e., a real monochromatic scalar field is represented by

$$\phi(x, y, z, t) = \hat{\psi}(x, y, z) e^{-j2\pi\nu_0 t} + \hat{\psi}^*(x, y, z) e^{j2\pi\nu_0 t} \quad (76)$$

whose transform is (with respect to time)

$$\hat{\phi}(x, y, z, \nu_0) = \hat{\psi}(x, y, z) \delta(\nu - \nu_0) + \hat{\psi}^*(x, y, z) \delta(\nu + \nu_0) \quad (77)$$

It is verified that $\hat{\phi}$ given by (77) satisfies the scalar Helmholtz wave eqn. (75) if $\hat{\psi}(x, y, z)$ satisfies the scalar Helmholtz wave equation, i.e.,

$$[\nabla^2 + (2\pi)^2 k^2 \hat{n}^2] \hat{\psi}(x, y, z) = 0 \quad (78)$$

where: $\hat{\psi}(x,y,z)$ is the complex spatial amplitude function for the monochromatic scalar wave $\hat{\psi}(x,y,z,v_0)$.

Thus for the monochromatic case, solution to (75) is accomplished by finding a solution to (78) for $\hat{\psi}(x,y,z)$, then substituting into (76). Approximation methods are found in the literature which typically characterize the index of refraction n as follows

$$n(x,y,z) = n_0 + n_1(x,y,z) \quad (79)$$

where $n_0 \gg n_1$

n_0 is the turbulence free refractive index of the atmosphere and n_1 is the fluctuating component of n . Further it is assumed that $n_1(x,y,z)$ is a zero-mean, gaussian random variable so that it is completely characterized by its second moment, or autocorrelation function.

1. Method of small perturbations

From equations (78) and (79) where n_0 is set equal to unity and using $k_0 = 2\pi k$ we have

$$\nabla^2 \psi + k_0^2 (1 + n_1)^2 \psi = 0 \quad (80)$$

A solution $\psi = \psi_0 + \psi_1$ is sought where ψ_0 is the unperturbed portion of the wave solution and satisfies $\nabla^2 \psi_0 + k_0^2 \psi_0 = 0$. Substituting $\psi = \psi_0 + \psi_1$ into (80) and clearing gives

$$\nabla^2 \psi_1 + k_0^2 \psi_1 + 2n_1 k_0^2 (\psi_0 + \psi_1) + k_0^2 n_1^2 (\psi_0 + \psi_1) = 0 \quad (81)$$

The last term in (81) is of order n_1^2 and is neglected; further the method of small perturbations assumes

$$|\psi_1| \ll |\psi_0| \quad (82)$$

so that equation (81) is written as

$$\nabla^2 \psi_1 + k_0^2 \psi_1 = -2n_1 k_0^2 \psi_0 \quad (83)$$

Solutions to the unperturbed wave equation have the form

$$\psi_0 = A_0 e^{iS_0}$$

where A_0 and S_0 are the amplitude and phase.

Then if we express the solution $\psi = \psi_0 + \psi_1$ in the form

$$\psi = \psi_0 + \psi_1 = A e^{iS}$$

then Tatarski shows that for $|\psi_1| \ll |\psi_0|$ we have

$$\log \psi = \log A + iS = \log A_0 + iS_0 + \frac{\psi_1}{\psi_0}$$

equating real and imaginary terms we get

$$\begin{aligned} \log \frac{A}{A_0} &= \operatorname{Re} \left\{ \frac{\psi_1}{\psi_0} \right\} = x \\ S - S_0 &= \operatorname{Im} \left\{ \frac{\psi_1}{\psi_0} \right\} = S_1 \end{aligned} \quad (84)$$

Thus we have

$$\begin{aligned} \psi_1 &= e^{x + iS_1} = \log x e^{iS_1} \\ &= A_1 e^{iS_1} \end{aligned} \quad (85)$$

where:

$$\psi_1(r) = \frac{k_0^2}{2\pi\psi_0(r)} \iiint \frac{n_1(r') \psi_0(r') e^{ik_0|r-r'|}}{|r-r'|} dr' \quad (86)$$

is the small perturbation solution to (83) in terms of n_1 and ψ_0 , where $r = (x, y, z)$.

$\psi_0(r)$ which must be known to solve equation (86) is easily found for plane monochromatic waves, i.e. for a wave propagating in the z direction

$$\psi_0(r) = A_0 e^{ik_0 z} \quad (87)$$

and (86) becomes

$$\psi_1(r) = \frac{k_0^2}{2\pi} \iiint n_1(r') e^{-ik_0(z-z')} \frac{e^{ik_0|r-r'|}}{|r-r'|} dr' \quad (88)$$

Further simplifications to (88) are made based on small scattering angle assumptions.

2. The Rytov method

This method uses a different approach to solving (80) which, instead of requiring smallness of ψ_1 compared to ψ_0 , uses the less restrictive condition that spatial fluctuations in phase and amplitude due to refractive index fluctuations be slowly varying with respect to the optical wavelength; that is,

$$\begin{aligned} |\nabla S_1| &\ll 2\pi \\ |\nabla \chi| &\ll 1 \end{aligned} \quad (89)$$

The approach is to find a solution ψ to (80) where

$$\psi = e^{\chi + iS} = e^{B(r)} \quad (90)$$

where $B(r) = B_0(r) + B_1(r)$

and $|\nabla B_1| \ll |\nabla B_0|$.

It can be shown by substituting (90) into (80) and e^{B_0} into (80) that B_0 satisfies the vacuum equation

$$\nabla^2 B_0 + (\nabla B_0)^2 + k_0^2 = 0 \quad (91)$$

and that B_1 satisfies approximately

$$\nabla^2 B_1 + 2\nabla B_0 \cdot \nabla B_1 + 2k_0^2 n_1 = 0 \quad (92)$$

The solution to (92) is given by the form in (86) so that the method of small perturbations and the Rytov method give the same results. However, The Rytov method has a wider range of validity. Hufnagel and Stanley point out that even the Rytov method is based on an assumption which fails for $|B| \approx 1$. Further, they develop a solution based on a statistical approach which gives the mutual coherence function without finding an explicit solution for ψ .

3. The Markov-parabolic approximation

Starting with an optical signal propagating in the Z direction

$$\tilde{E} = \psi(x, y, z) e^{i(k_0 z - \omega t)} \quad (93)$$

the wave equation (80) is then approximated by

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + 2ik_0 \frac{\partial \psi}{\partial z} + 2k_0^2 n_1 \psi = 0 \quad (94)$$

This is the so-called parabolic approximation to the wave equation and is applicable if the beam spread angle for the propagating wave is small.

In other words with respect to the succeeding figure we have

$$|r-r'| = \sqrt{(x-x')^2 + (y-y')^2 + (z')^2} \quad (95)$$

where $|x-x'| \ll z'$

$$|y-y'| \ll z'$$

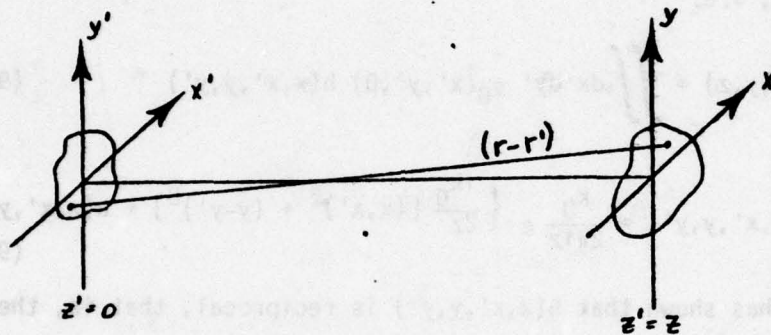


Figure 3. Small-angle scattering.

The solution to (94) can be written as [13]

$$\psi(x,y,z) = \frac{k_0}{2\pi iz} \iint dx' dy' \psi_0(x',y',0) \cdot e^{\left\{ \frac{ik_0}{2z} [(x-x')^2 + (y-y')^2] + u(x,x',y,y') \right\}} \quad (96)$$

where $\psi_0(x',y',0)$ is the field in the $z'=0$ plane, $\psi(x,y,z)$ is the field in the $z'=z$ plane, and $u(x,x',y,y')$ is the random part of the complex phase of a spherical wave propagating from point $(x',y',0)$ to point (x,y,z) . Equation (96) without the dependence on $u(x,x',y,y')$ is simply the Huygen-Fresnel formula which is an approximate solution to the scalar

Helmholtz wave equation in a homogeneous medium. Thus the result in (96) can be considered to be an extension of the Huygen-Fresnel formula to include media with small-scale random fluctuations. Additionally, (96) is conveniently expressed in terms of linear systems theory since it relates fields in the input plane $(x', y', z'=0)$ and in the output plane (x, y, z) , i.e.

$$\psi(x, y, z) = \iint dx' dy' \psi_0(x', y', 0) h(x, x', y, y') \quad (97)$$

$$\text{where } h(x, x', y, y') = \frac{k_0}{2\pi iz} e. \left\{ \frac{ik_0}{2z} [(x-x')^2 + (y-y')^2] + u(x, x', y, y') \right\} \quad (98)$$

Shapiro [24] has shown that $h(x, x', y, y')$ is reciprocal, that is, the input and output planes can be reversed and (97) still holds. Using Markov approximation methods, the moments of $\psi(x, y, z)$ can be found to yield the spatial coherence function of the propagating field.

$$M(\rho_1, \rho_2, z) = \langle \psi(\rho_1, z) \psi^*(\rho_2, z) \rangle \quad (99)$$

where M is the spatial coherence function at $z'=z$, the $\langle \rangle$ brackets denote an ensemble average and $\rho_i = (x_i, y_i)$. Using (97) in (99) gives

$$M(\rho_1, \rho_2, z) = \iint dx_1' dy_1' \iint dx_2' dy_2' \psi_0(x_1', y_1', 0) \psi_0^*(x_2', y_2', 0) \langle h(x_1, y_1, x_1', y_1') h^*(x_2, y_2, x_2', y_2') \rangle \quad (100)$$

where for the homogeneous medium, $\langle h(\rho_1, \rho_1') h^*(\rho_2, \rho_2') \rangle$ is easily shown to be

$$\left(\frac{k_0}{2\pi z} \right)^2 \exp \left\{ \frac{ik_0}{2z} [(\rho_1 - \rho_1')^2 - (\rho_2 - \rho_2')^2] \right\} \quad (101)$$

and for the fluctuating medium is shown not so easily [6] to be

$$\begin{aligned} \langle h(\rho_1, \rho_1') h^*(\rho_2, \rho_2') \rangle &= \left(\frac{k_0}{2\pi z} \right)^2 \exp \left\{ \frac{ik_0}{2z} [(\rho_1 - \rho_1')^2 - (\rho_2 - \rho_2')^2] \right. \\ &\quad \left. - \frac{\pi k_0^2}{4} \int_0^z dz' H \left[z', \frac{z'}{z} (\rho_1 - \rho_2) + (\rho_1' - \rho_2') \left(1 - \frac{z'}{z} \right) \right] \right\} \end{aligned} \quad (102)$$

$$\text{where } H(z, \xi) = 1.88 C_n^2(z) |\xi|^{5/3} \left[1 - 0.805 \left(\frac{|\xi|}{L_0} \right)^{1/3} \right] \quad (103)$$

C_n^2 is the refractive index structure coefficient

L_0 is a measure of the largest distance over which fluctuations in the refractive index are still considered to be correlated.

Another way of looking at what equation (100) means is given by recognizing that the function

$$M(\rho_1', \rho_2', 0) = \langle \psi(\rho_1', 0) \psi^*(\rho_2', 0) \rangle \quad (104)$$

is a measure of the spatial coherence in the input plane and equation (100) then gives the propagation of this coherence function through a medium described by $h(x, y, x', y')$ which is the point response for the medium. That $h(x, y, x', y')$ gives the response at (x, y, z) to a point source at $(x', y', 0)$. The relationship of spatial coherence to resolution or MTF is typically derived in terms of the Fourier transform of the intensity distribution in the focal plane of a simple lens which is illuminated by an atmospherically distorted plane wavefront. The problem with this approach is that for the short-exposure case, effects of the atmosphere are not separable from effects of the lens. A development by Fried [15] is given in Section 4 for MTF effects of the atmosphere for both long and short-term exposure.

4. The Huygen-Fresnel method

The Green's function method [25] is used to find a solution to (78). Consider a volume V enclosed by surface S . If $\hat{\psi}$ has continuous 1st and 2nd order spatial partial derivatives within V and on S ; further, if G is any other function which satisfies the same continuity conditions on $\hat{\psi}$, then Green's theorem yields

$$\iiint_V [\hat{\psi} \nabla^2 G - G \nabla^2 \hat{\psi}] dV = \iint_S [\hat{\psi} \frac{\partial G}{\partial n} - G \frac{\partial \hat{\psi}}{\partial n}] ds \quad (105)$$

GREEN'S THEOREM

n is derivative normal to S in outward direction. Now for a propagating optical field, we are interested in solving the boundary-value problem, given the field $\hat{\psi}(x,y,0)$ find $\hat{\psi}(x,y,z)$, where the field is propagating in the $+z$ direction. Thus for this problem, we define S to be the $z=0$ plane closed by an infinite radius surface such that V is the $z>0$ half sphere.

In (105), G is a suitable Green's function for the Helmholtz equation or since the Green's function is the response to a point source driving function, we have for the Helmholtz eqn.

$$[\nabla^2 + (2\pi)^2 k^2 \hat{n}^2] G(x,y,z,\xi,\eta,\zeta) = \delta(x-\xi)\delta(y-\eta)\delta(z-\zeta) \quad (106)$$

Using (78) and (106) in (105) gives

$$\begin{aligned} & \iiint_V [\hat{\psi}(\xi,\eta,\zeta) \{-(2\pi)^2 k^2 \hat{n}^2 G(x,y,z,\xi,\eta,\zeta) + \delta(x-\xi)\delta(y-\eta)\delta(z-\zeta)\} \\ & \quad - G(x,y,z,\xi,\eta,\zeta) \{-(2\pi)^2 k^2 \hat{n}^2 \hat{\psi}(\xi,\eta,\zeta)\}] dV \\ &= \iint_S [\hat{\psi}(\xi,\eta,0) [-\frac{\partial G(x,y,z,\xi,\eta,\zeta)}{\partial \zeta}]_{\zeta=0} \\ & \quad - G(x,y,z,\xi,\eta,0) [-\frac{\partial \hat{\psi}(\xi,\eta,\zeta)}{\partial \zeta}]_{\zeta=0}] d\xi d\eta \quad (107) \end{aligned}$$

where $\iint_S = \iint_{\substack{z=0 \text{ plane} \\ \text{or } S'}} + \iint_R$

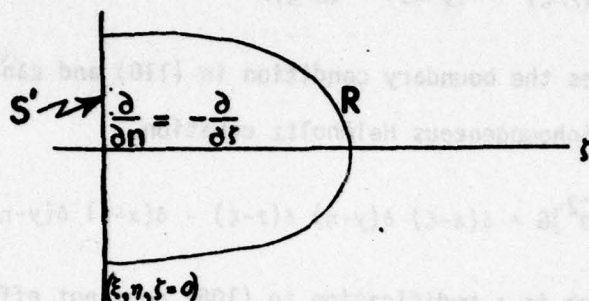


Figure 4. Surface for Green's function integral.
 * see justification on pp 379-380 in Born & Wolf,

Principles of Optics [16]. The left side of (107) simplifies to give

$$\iiint_V \hat{\psi}(\xi, \eta, \zeta) \delta(x-\xi, y-\eta, z-\zeta) dV = \hat{\psi}(x, y, z) \quad (108)$$

so that (107) becomes

$$\hat{\psi}(x, y, z) = -\iint_{S'} \left\{ \hat{\psi} \frac{\partial G}{\partial \zeta} - G \frac{\partial \hat{\psi}}{\partial \zeta} \right\} \Big|_{\zeta=0} d\xi d\eta \quad (109)$$

Or if the Green's function is chosen so that

$$G(x, y, z, \xi, \eta, 0) = 0 \quad (110)$$

then (109) becomes

$$\hat{\psi}(x, y, z) = -\iint \hat{\psi}(\xi, \eta, 0) \frac{\partial G(x, y, z, \xi, \eta, \zeta)}{\partial \zeta} \Big|_{\zeta=0} d\xi d\eta \quad (111)$$

(111) is the desired result. It gives the field at Z in terms of the field at Z=0 and the Green's function $G(x, y, z, \xi, \eta, \zeta)$, where G must satisfy (106) and (110). Thus it remains to find the Green's function.

Fortunately, the Green's function for the Helmholtz equation with the given boundary conditions above has been found already and is given by [26]

$$G(x, y, z, \xi, \eta, \zeta) = \frac{-e^{j2\pi k\hat{n}|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|} + \frac{e^{j2\pi k\hat{n}|\vec{r}-\vec{r}''|}}{4\pi|\vec{r}-\vec{r}''|} \quad (112)$$

where $|\vec{r}-\vec{r}'| = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2}$

and $|\vec{r}-\vec{r}''| = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z+\zeta)^2}$

Clearly (112) satisfies the boundary condition in (110) and can be shown to satisfy the inhomogeneous Helmholtz equation

$$[\nabla^2 + 4\pi^2 k^2 n^2]G = \delta(x-\xi) \delta(y-\eta) \delta(z-\zeta) - \delta(x-\xi) \delta(y-\eta) \delta(z+\zeta) \quad (113)$$

This result (113) which is a modification to (106) does not effect the results in (107) and (108) because the second delta function is outside the volume of integration V , and thus contributes nothing to the integral.

Further, it is possible to show that from (112)

$$\left. \frac{\partial G(x,y,z,\xi,\eta,\zeta)}{\partial \zeta} \right|_{\zeta=0} = \frac{\partial}{\partial z} \frac{e^{j2\pi k n [(x-\xi)^2 + (y-\eta)^2 + z^2]^{1/2}}}{2\pi [(x-\xi)^2 + (y-\eta)^2 + z^2]^{1/2}} \quad (114)$$

Then using (114) in (111) we have the solution $\hat{\psi}(x,y,z)$

$$\hat{\psi}(x,y,z) = - \iint_S \hat{\psi}(\xi,\eta,0) \frac{\partial}{\partial z} \left\{ \frac{e^{j2\pi k n [(x-\xi)^2 + (y-\eta)^2 + z^2]^{1/2}}}{2\pi [(x-\xi)^2 + (y-\eta)^2 + z^2]^{1/2}} \right\} d\xi d\eta \quad (115)$$

or

$$\hat{\psi}(x,y,z) = \iint d\xi d\eta \hat{g}(x-\xi,y-\eta,z) \hat{\psi}(\xi,\eta,0) \quad (116)$$

where

$$\hat{g}(x,y,z) = -\frac{\partial}{\partial z} \left\{ \frac{e^{j2\pi k n [x^2+y^2+z^2]^{1/2}}}{2\pi [x^2+y^2+z^2]^{1/2}} \right\} \quad (117)$$

Equations (116) and (117) are the desired result and define the Rayleigh-Sommerfeld Integral. If we perform the derivative indicated in (117)

where $s = [x^2 + y^2 + z^2]^{1/2}$ then

$$-\frac{\partial}{\partial z} \left\{ \frac{e^{j2\pi k \hat{n} s}}{2\pi s} \right\} = -\frac{j k \hat{n} z}{s^2} e^{j2\pi k \hat{n} s} \left[1 - \frac{1}{j2\pi k \hat{n} s} + \frac{s^2}{\hat{n} z} \frac{d\hat{n}}{dz} \right] \quad (118)$$

For optical frequencies $\frac{d\hat{n}}{dz}$ is of the order 10^{-6} . Thus for observation ranges less than 10^5 meters, the last term in brackets in (118) is much less than unity and is neglected. Further for $s \gg \lambda$, $ks \gg 1$ and the second term in brackets is negligible. Thus we have the modified Rayleigh-Sommerfeld result.

$$\hat{\psi}(x, y, z) = \iint d\xi d\eta \frac{\hat{n}}{j\lambda} \left(\frac{e^{j2\pi k \hat{n} r}}{r} \right) \cos(z, r) \hat{\psi}(\xi, \eta, 0) \quad (119)$$

Modified Rayleigh-Sommerfeld integral for linear, isotropic, weakly inhomogeneous, weakly absorbing medium ($r \gg \lambda$)

where: $\cos(z, r) = \frac{z}{r}$ = cosine between z and r and $r = \sqrt{(x-\xi)^2 + (y-\eta)^2 + z^2}$

Equation (119) is further modified by the Fresnel approximation which holds for source and observation points close to the z axis, i.e.

$$\sqrt{(x-\xi)^2 + (y-\eta)^2} \ll z = d \quad (120)$$

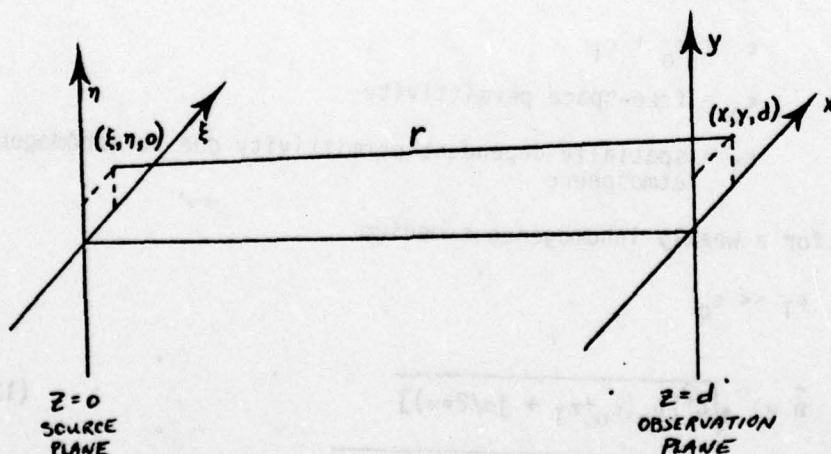


Figure 5. Fresnel region.

Then the following approximations follow

$$\begin{aligned}\cos(z, r) &\approx 1 \\ r &= d \text{ for amplitude term} \\ r &= d + \frac{(x-\xi)^2}{2d} + \frac{(y-n)^2}{2d} + \dots \text{ for phase term}\end{aligned}\quad (121)$$

Using (121) in (119) gives the Fresnel Approximation

$$\hat{\psi}(x, y, d) = \iint d\xi dn \left(\frac{\hat{n}e}{j\lambda d} \right)^{j2\pi k\hat{n}d} e^{\frac{j2\pi k\hat{n}}{d} [(x-\xi)^2 + (y-n)^2]} \cdot \hat{\psi}(\xi, n, 0) \quad (122)$$

FRESNEL APPROXIMATION

Examination of \hat{n} , which is a slowly varying function of spatial coordinates, yields the following

$$\hat{n} = c\sqrt{\mu\epsilon} = c\sqrt{\mu\left(\epsilon + \frac{j\sigma}{2\pi\nu}\right)} = \sqrt{c^2\left[\mu\left(\epsilon + \frac{j\sigma}{2\pi\nu}\right)\right]} \quad (123)$$

If assume that the magnetic permeability μ and the conductivity σ are essentially constant (not dependent on spatial coordinates), then the only dependence on spatial coordinates is in ϵ . Then we can define

$$\begin{aligned}\mu &= \mu_0 = \text{permeability of free space} \\ \sigma &= \text{constant} \\ \epsilon &= \epsilon_0 + \epsilon_1 \\ \epsilon_0 &= \text{free-space permittivity} \\ \epsilon_1 &= \text{spatially dependent permittivity due to inhomogeneous atmosphere}\end{aligned}$$

Further, for a weakly inhomogeneous medium

$$\epsilon_1 \ll \epsilon_0$$

Now

$$\begin{aligned}\hat{n} &= \sqrt{c^2\left[\mu_0(\epsilon_0 + \epsilon_1 + j\sigma/2\pi\nu)\right]} \\ &= \sqrt{c^2\mu_0\epsilon_0\left(1 + \frac{\epsilon_1}{\epsilon_0} + j\frac{\sigma}{2\pi\nu\epsilon_0}\right)}\end{aligned}\quad (124)$$

or since $c^2 = \frac{1}{\nu_0 \epsilon_0}$ (124) reduces to

$$\hat{n} = \sqrt{1 + \left(\frac{\epsilon_1}{\epsilon_0} + j \frac{\sigma}{2\pi\nu\epsilon_0} \right)} \quad (125)$$

which is of the form $1 + \hat{C}$, where \hat{C} is a complex number with magnitude much less than one. The square root is approximated by

$$\sqrt{1 + \hat{C}} \approx 1 + \hat{C} \quad (126)$$

Using (126) in (125) yields

$$\hat{n} \approx 1 + \frac{\epsilon_1}{\epsilon_0} + j \frac{\sigma}{2\pi\nu\epsilon_0} \quad \begin{array}{l} \text{COMPLEX} \\ \text{REFRACTIVE} \\ \text{INDEX} \end{array} \quad (127)$$

If we substitute (127) into the Modified Rayleigh-Sommerfeld integral (119) according to the following:

- for the multiplicative amplitude term in (119) use $\hat{n} \sim n_0 = 1$
- for the phase term involving \hat{n}_1 use (127)

Then

$$\hat{\psi}(x,y,z) \approx \iint d\xi d\eta \left(\frac{e^{j2\pi kr}}{j\lambda r} \right) e^{\frac{-k\sigma r}{\nu\epsilon_0}} e^{\frac{j2\pi k\epsilon_1 r}{\epsilon_0}} \cos(z,r) \hat{\psi}(\xi,\eta,0) \quad (128)$$

or if we define

$$\alpha = \frac{2k\sigma}{\nu\epsilon_0} = \text{atmospheric extinction coefficient}$$

$$\theta(r) = \frac{2\pi k\epsilon_1 r}{\epsilon_0} = \text{turbulence induced phase shift (recall } \epsilon_1 \text{ is also a function of } r)$$

$$\hat{\psi}(x,y,z) \approx \iint d\xi d\eta e^{-\frac{\alpha r}{2}} e^{j\theta(r)} \left(\frac{e^{j2\pi kr}}{j\lambda r} \right) \cos(z,r) \hat{\psi}(\xi,\eta,0) \quad (129)$$

Equation (129) is the modified Rayleigh-Sommerfeld integral extended to include atmospheric turbulence and absorption. It gives the complex

scalar amplitude of an optical field at $z = z$ in terms of the known amplitude at $z = 0$. Now using the Fresnel approximation as defined in (120) and (121) applied to (129) gives

$$\hat{\psi}(x,y,d) \approx \iint d\xi d\eta e^{-\frac{\alpha d}{2}} e^{j\theta(r)} \left(\frac{e^{j2\pi kd}}{j\lambda d} \right) e^{j\frac{\pi k}{d}[(x-\xi)^2 + (y-\eta)^2]} \cdot \hat{\psi}(\xi,\eta,0)$$

or pulling out the terms not dependent on ξ and η

$$\hat{\psi}(x,y,d) \approx \frac{e^{j2\pi kd}}{j\lambda d} e^{-\frac{\alpha d}{2}} \iint d\xi d\eta e^{j[\frac{\pi k}{d}\{(x-\xi)^2 + (y-\eta)^2\} + \theta(r)]} \cdot \hat{\psi}(\xi,\eta,0)$$

(130)

Equation (130) is the extended Fresnel approximation including atmospheric turbulence and absorption.

IV. IMAGING THROUGH THE ATMOSPHERE

In this section, the results of Section 3.0 are applied to the imaging sensor problem.

Other investigators have looked at various portions of this problem. Lutomirski and Yura [13] give the development for the extended Huygen-Fresnel formula. In [27] Yura develops a general expression for the mutual coherence function of a finite optical beam propagating in a turbulent medium, and in [28] extends the results to a sea water medium by including an absorption term.

A later paper by Lutomirski and Yura [12] goes even further in bringing together all the necessary ingredients for a comprehensive theory on the effects of the atmosphere on imaging of real objects. The development is, however, still monochromatic; and even though an expression is developed for modulation contrast, its usefulness is limited by choosing a sinusoidal object function as an example. Thus the contrast found is really MTF (i.e. contrast transfer at one spatial frequency).

A similar development for the MTF through sea water is given by Ishimaru [11].

As with most journal papers, the above references omit considerable detail in their technical development. Thus one goal of the present effort was to present a comprehensive theory with essentially all the steps included.

The extended Huygen-Fresnel Integral, eqn. (68), gives the complex scalar optical amplitude at z in terms of the field at $z = 0$ and the atmospheric point response function. Next, irradiance levels are computed from the magnitude-squared of the complex amplitude function. From this can be found the point-spread function and modulation transfer function (MTF). Then by including the effects of ambient irradiance it is possible to compute contrast levels at both the object and observation planes. Further, since the approach for computing contrast has been developed in terms of spatial amplitude functions, the dependence on target size and range is included in the result.

In (130) it is pointed out that $\theta(r)$ is a random phase perturbation caused by inhomogeneities in the atmospheric refractive index along the path between source and observation points. As a random phenomenon, quantitative performance measures based on (130) must be ensemble averaged.

First order statistics of $\theta(r)$ - recall from the definition of $\theta(r)$

$$\theta(r) = \frac{2\pi k \epsilon_1}{\epsilon_0} r = \theta(x, y, d, \xi, \eta)$$

where ϵ_1 is the random quantity. Further ϵ_1 is a small perturbation about unity and may be assumed to be zero mean. Thus the expected value

$$\langle \theta(r) \rangle = 0 \quad (131)$$

Second order statistics of $\hat{\psi}(x, y, d)$ - since all photodetectors respond to radiance averaged over many wavelengths, our interest is in the quantity (for long viewing times)

$$\hat{E}(x, y, d) = \langle \hat{\psi}(x, y, d) \hat{\psi}^*(x, y, d) \rangle \quad (132)$$

Then from (130) for the Fresnel approximation

$$\hat{E}(x,y,d) = \frac{e^{-\alpha d}}{\lambda^2 d^2} \iiint d\xi d\eta d\xi' d\eta' \hat{\psi}(\xi,\eta,0) \hat{\psi}^*(\xi',\eta',0) e^{j\frac{\pi k}{d} \{(x-\xi)^2 + (y-\eta)^2 - (x-\xi')^2 - (y-\eta')^2\}} \langle e^{j[\theta(r) - \theta(r')]} \rangle \quad (133)$$

where: $r = \sqrt{(x - \xi)^2 + (y - \eta)^2 + d^2}$
 $r' = \sqrt{(x - \xi')^2 + (y - \eta')^2 + d^2}$

For an incoherent source

$$\hat{\psi}(\xi,\eta,0) \hat{\psi}^*(\xi',\eta',0) = |\hat{\psi}(\xi,\eta,0)|^2 \delta(\xi-\xi', \eta-\eta') \quad (134)$$

Using (134) in (133) gives

$\hat{E}(x,y,d) = \frac{e^{-\alpha d}}{\lambda^2 d^2} \iint d\xi d\eta \hat{E}(\xi,\eta,0)$	(135)
Irradiance transfer through the atmosphere	

where $\hat{E}(\xi,\eta,0) = |\hat{\psi}(\xi,\eta,0)|^2$

Equation (135) is essentially Huygen's Principle with application to irradiance. That is, for the unfocused incoherent source at $(\xi,\eta,0)$, the irradiance at (x,y,d) is simply a summation of spherical wavelets emanating from all source points at approximate distance d and attenuated by the atmosphere. The average irradiance of such a wave is unaffected by atmospheric turbulence. This result is particularly interesting in light of the fact that turbulence very definitely affects a focused wave-front. Further, it will affect the irradiance from a source that exhibits partial coherence spatially.

Note that the ensemble average in (132) implies a viewing time long

enough that the observed irradiance is accurately represented by an ensemble average. Further, the interchanging of time and ensemble averages implies spatial ergodicity for the fluctuating refractive index term which further says that the atmospheric refractive index is a shift-invariant random quantity. For the Fresnel approximation, the assumption of shift invariance implies that the total object-observation volume consisting of a long narrow cylindrical region between $z=0$, and $z = d$ is a region of isoplanatism (i.e., shift-invariant).

Most references which develop equations showing the effect of the atmosphere on optical signals assume that the object-observation volume is essentially isoplanatic ([12], [27] - [29] and others.)

A. Simple Lens - Atmosphere Imaging System

Next we consider the effects of the atmosphere on optical signals received by a simple imaging system. Consider the system shown in Figure 6.

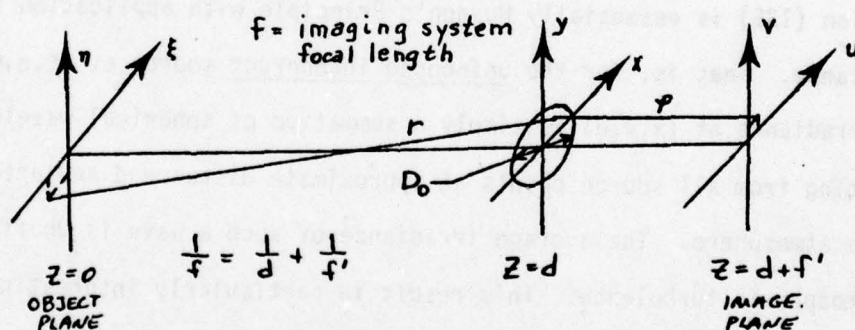


Figure 6. Simple atmosphere-lens imaging system.

It is desired to find $\hat{\psi}(u, v, f')$, the optical complex amplitude in the lens image plane, in terms of the input amplitude $\hat{\psi}(\xi, \eta, 0)$ and parameters describing the lens and atmospheric path.

Starting with the extended, modified Rayleigh-Sommerfeld integral given by (129) we have

$$\hat{\psi}(x,y,d) = \iint d\xi d\eta e^{-\frac{\alpha r}{2}} e^{j\theta(r)} \left(\frac{e^{j2\pi kr}}{j\lambda r} \right) \hat{\psi}(\xi,\eta,0) \quad (136)$$

where we have used $\cos(z,r) \approx 1$ for the lens diameter, D_0 , much smaller than d .

If the lens transmission function is represented by $\hat{t}(x,y)$ then we have at the lens output plane an optical signal given by $\hat{t}(x,y) \hat{\psi}(x,y,d)$. Further, we assume this optical signal propagates to the image plane under vacuum conditions (i.e., the path between lens and image plane is linear, isotropic, non-absorbing, source-free and homogeneous). Thus propagation from the lens to image plane is governed by the vacuum, modified Rayleigh-Sommerfeld integral.

$$\hat{\psi}(u,v,f') = \iint dx dy \left(\frac{e^{j2\pi k\rho}}{j\lambda\rho} \right) \cos(z,\rho) \hat{t}(x,y) \hat{\psi}(x,y,d) \quad (137)$$

Image plane complex amplitude

where: $\rho = \sqrt{(u-x)^2 + (v-y)^2 + (f')^2}$
 and for $f' \gg \sqrt{(u-x)^2 + (v-y)^2}$; $\cos(z,\rho) \approx 1$
 further, as shown in Appendix D, for a thin lens

$$\hat{t}(x,y) = \hat{p}(x,y) e^{-\frac{j\pi k(x^2 + y^2)}{f}} \quad (138)$$

where $\hat{p}(x,y)$ is the pupil function given by

$$\hat{p}(x,y) = w(x,y) e^{j2\pi(n_L-1)d_0k} = w(x,y) e^{jA_L} ; \quad x^2 + y^2 < \left(\frac{D_0}{2}\right)^2$$

$$= 0 ; \quad x^2 + y^2 > \left(\frac{D_0}{2}\right)^2$$

where: n_L is the lens refractive index, A_L is a constant, d_0 is the lens thickness on its optical axis and $\hat{t}(x,y)$ gives the additional phase shift through the lens plus includes windowing effects from the lens aperture function, $w(x,y)$. Using (136) and (139) in (137) gives

$$\hat{\psi}(u,v,f') = \iint dx dy \iint d\xi d\eta \left(\frac{e^{j2\pi k r}}{j\lambda r} \right) e^{-\frac{\alpha r}{2}} e^{j\theta(r)} \cdot \hat{\psi}(\xi,\eta,0) \cdot \left(\frac{e^{j2\pi k \rho}}{j\lambda \rho} \right) \hat{p}(x,y) e^{-\frac{j\pi k(x^2+y^2)}{f}} \quad (140)$$

Equation (140) is the extended, modified Rayleigh-Sommerfeld integral which gives the image plane complex optical amplitude $\hat{\psi}(u,v,f')$ in terms of the object plane amplitude $\hat{\psi}(\xi,\eta,0)$ as viewed through a slightly inhomogeneous, slightly absorbing atmospheric path.

Next we consider (140) under the Fresnel approximation, i.e., the following apply

for amplitude terms

$$\begin{aligned} \frac{1}{r} &= \frac{1}{\sqrt{(x-\xi)^2 + (y-\eta)^2 + d^2}} \approx \frac{1}{d} \\ \frac{1}{\rho} &= \frac{1}{\sqrt{(u-x)^2 + (v-y)^2 + (f')^2}} \approx \frac{1}{f'} \approx \frac{1}{f} \\ e^{-\frac{\alpha r}{2}} &\approx e^{-\frac{\alpha d}{2}} \quad \text{absorption term} \end{aligned} \quad (141)$$

for phase terms

$$\begin{aligned} r &= \sqrt{(x-\xi)^2 + (y-\eta)^2 + d^2} \approx d + \frac{(x-\xi)^2}{2d} + \frac{(y-\eta)^2}{2d} \\ \rho &= \sqrt{(u-x)^2 + (v-y)^2 + (f')^2} \approx f' + \frac{(u-x)^2}{2f'} + \frac{(v-y)^2}{2f'} \end{aligned} \quad (142)$$

Then (140) is approximated by

$$\begin{aligned} \hat{\psi}(u,v,f') = & e^{-\frac{\alpha d}{2}} e^{j2\pi k d} e^{j2\pi k f'} \iint dx dy \iint d\xi d\eta \\ & \cdot e^{j\theta(x,y,d,\xi,\eta)} \frac{j\pi k}{d} [(x-\xi)^2 + (y-\eta)^2] \hat{\psi}(\xi,\eta,0) \frac{j\pi k}{f'} [(u-x)^2 + (v-y)^2] \\ & \cdot \hat{p}(x,y) e^{-\frac{j\pi k(x^2+y^2)}{f}} \end{aligned} \quad (143)$$

To shorten the notation we make use of the vector two-dimensional forms

$$\begin{aligned} \bar{x} &= (x,y) \\ \bar{\xi} &= (\xi,\eta) \\ \bar{u} &= (u,v) \end{aligned} \quad (144)$$

Also, making use of (139) and the constant phase terms in (143) we re-write (143) as

$$\begin{aligned} \hat{\psi}(\bar{u},f') = & e^{-\alpha d/2} e^{jA} \iint d\bar{x} \iint d\bar{\xi} e^{j\theta(\bar{x},\bar{\xi},d)} \hat{\psi}(\bar{\xi},0) \\ & \cdot \frac{j\pi k}{d} |\bar{x}-\bar{\xi}|^2 \frac{j\pi k}{f'} |\bar{u}-\bar{x}|^2 w(\bar{x}) e^{-j\frac{\pi k \bar{x} \cdot \bar{x}}{f}} \end{aligned} \quad (145)$$

where: $A = 2\pi k[(d+f') + (n_L - 1)d_0]$

and $w(\bar{x}) = 1 ; |\bar{x}| < D_0/2$

$0 ; |\bar{x}| > D_0/2$

Equation (145) gives the Fresnel approximation for the image plane optical field $\hat{\psi}(\bar{u},f')$ in terms of an object field $\hat{\psi}(\bar{\xi},0)$ imaged through the atmosphere, including absorption. Recall that for the model as developed to this point $\theta(\bar{x},\xi,d)$ is the random phase induced by small fluctuations in refractive index along the optical path, and is the only random

quantity in (145). Strictly speaking the atmospheric extinction coefficient, α , is also a slowly varying random function of the optical path.

B. Point Spread Function and Modulation Transfer Function

To find the irradiance in the image plane we must find

$$\hat{E}(\bar{u}, f') = \langle \hat{\psi}(\bar{u}, f') \hat{\psi}^*(\bar{u}, f') \rangle \quad (146)$$

Using (145) in (146)

$$\begin{aligned} \hat{E}(\bar{u}, f') = & \frac{e^{-\alpha d}}{\lambda^4 d^2 f'^2} \left[\iint d\bar{x} \iint d\bar{x}' \iint d\bar{\xi} \iint d\bar{\xi}' \hat{\psi}(\bar{\xi}, 0) \hat{\psi}^*(\bar{\xi}', 0) \right. \\ & \cdot W(\bar{x}) W(\bar{x}') e^{j\frac{\pi k}{d}(|\bar{x}-\bar{\xi}|^2 - |\bar{x}'-\bar{\xi}'|^2)} e^{j\frac{\pi k}{f'}(|\bar{u}-\bar{x}|^2 - |\bar{u}-\bar{x}'|^2)} \\ & \left. \cdot e^{-j\frac{\pi k}{f}(\bar{x} \cdot \bar{x} - \bar{x}' \cdot \bar{x}')} e^{j[\theta(\bar{x}, \bar{\xi}, d) - \theta(\bar{x}', \bar{\xi}', d)]} \right] \quad (147) \end{aligned}$$

Again for an incoherent source we have

$$\hat{\psi}(\bar{\xi}, 0) \hat{\psi}^*(\bar{\xi}', 0) = |\hat{\psi}(\bar{\xi}, 0)|^2 \delta(\bar{\xi} - \bar{\xi}') \quad (148)$$

which physically says that the irradiance at a given point depends only on the complex amplitude at that point. The result of (148) on (147) is that only $\bar{\xi} = \bar{\xi}'$ contributes to the integral and (147) reduces to

$$\begin{aligned} \hat{E}(\bar{u}, f') = & \frac{e^{-\alpha d}}{\lambda^4 d^2 f'^2} \iint d\bar{x} \iint d\bar{x}' \iint d\bar{\xi} \hat{E}(\bar{\xi}, 0) w(\bar{x}) w(\bar{x}') \\ & \cdot e^{j\frac{\pi k}{d}(|\bar{x}-\bar{\xi}|^2 - |\bar{x}'-\bar{\xi}|^2)} e^{j\frac{\pi k}{f'}(|\bar{u}-\bar{x}|^2 - |\bar{u}-\bar{x}'|^2)} e^{-j\frac{\pi k}{f}(\bar{x} \cdot \bar{x} - \bar{x}' \cdot \bar{x}')} \\ & \cdot \langle e^{j[\theta(\bar{x}, \bar{\xi}, d) - \theta(\bar{x}', \bar{\xi}, d)]} \rangle \quad (149) \end{aligned}$$

Or we can write

$$\hat{E}(\bar{u}, f') = \iint d\bar{\xi} \hat{E}(\bar{\xi}, 0) G_1(\bar{u}, f', \bar{\xi}, 0) \quad (150)$$

where

$$G_1(\bar{u}, f', \bar{\xi}, 0) = \frac{e^{-ad}}{\lambda^2 d^2 f', 2} \iint d\bar{x} \iint d\bar{x}'$$

$$w(\bar{x})w(\bar{x}') \left\langle e^{j[\theta(\bar{x}, \bar{\xi}, d) - \theta(\bar{x}', \bar{\xi}, d)]} \right\rangle$$

$$\cdot e^{j\frac{\pi k}{d}(|\bar{x} - \bar{\xi}|^2 - |\bar{x}' - \bar{\xi}|^2)} e^{j\frac{\pi k}{f'}(|\bar{u} - \bar{x}|^2 - |\bar{u} - \bar{x}'|^2)} e^{-j\frac{k\pi}{f}(\bar{x} \cdot \bar{x} - \bar{x}' \cdot \bar{x}')} \quad (151)$$

G_1 is the ^{total}_A atmospheric-lens system Green's function. The expression in (151) can be simplified by recognizing the effect of certain functions in (151) and by combining the exponential terms, excluding those in the ensemble brackets $\langle \cdot \rangle$.

We have the combined exponents

$$j\pi k \left(\frac{|\bar{x} - \bar{\xi}|^2}{d} - \frac{|\bar{x}' - \bar{\xi}|^2}{d} + \frac{|\bar{u} - \bar{x}|^2}{f'} - \frac{|\bar{u} - \bar{x}'|^2}{f'} - \frac{\bar{x}^2 - \bar{x}'^2}{f} \right) \quad (152)$$

where we have used $\bar{x} \cdot \bar{x} = \bar{x}^2 = x^2 + y^2$ and $\frac{1}{f} = \frac{1}{f'} + \frac{1}{d}$, then by completing the squares and simplifying, (152) becomes

$$-j\frac{2\pi k}{d} [(\bar{x} - \bar{x}') \cdot (\bar{\xi} + \frac{d}{f'}, \bar{u})] \quad (153)$$

The term in ensemble brackets in (151) is recognized as the atmospheric mutual coherence function (MCF); [12], [13] and for an assumed isoplanatic

source-observation volume (i.e., shift invariant), the MCF does not depend on the source coordinates $\bar{\xi}$ and further depends only on the difference vector $(\bar{x}-\bar{x}')$. Thus we have for (150) and (151)

$$\hat{E}(\bar{u}, f') = \iint d\bar{\xi} \hat{E}(\bar{\xi}, 0) S(\bar{\xi} + \frac{d}{f}, \bar{u}) \quad (154)$$

Output irradiance as fcn of PSF and input irradiance

where $\hat{S}(\bar{\xi})$ is the atmosphere-lens system point spread function (PSF) and is given by:

$$\hat{S}(\bar{\xi}) = \frac{e^{-\alpha d}}{\lambda^4 d^2 f'^2} \iint d\bar{x} \iint d\bar{x}' w(\bar{x}) w(\bar{x}') \text{MCF}(\bar{x}-\bar{x}') \cdot e^{-j \frac{2\pi k}{d} (\bar{x}-\bar{x}') \cdot \bar{\xi}} \quad (155)$$

Point Spread Function For Atmosphere-Lens System

Further, if we make the following change of variable

$$\bar{\rho} = \bar{x} - \bar{x}'$$

$$\bar{R} = \frac{\bar{x} + \bar{x}'}{2} \quad (156)$$

then $\bar{x} = \bar{R} + \frac{\bar{\rho}}{2}; \bar{x}' = \bar{R} - \frac{\bar{\rho}}{2}$

Substituting (156) into (155) yields

$$\hat{S}(\bar{\xi}) = \frac{e^{-\alpha d}}{\lambda^4 d^2 f'^2} \iint d\bar{\rho} \iint d\bar{R} w(\bar{R} + \frac{\bar{\rho}}{2}) w(\bar{R} - \frac{\bar{\rho}}{2}) \cdot \text{MCF}(\bar{\rho}) e^{-j \frac{2\pi k \bar{\rho} \cdot \bar{\xi}}{d}} \quad (157)$$

Alternate Form of Point Spread Function

Now eqns. (154) and (157) give the relationship between source irradiance and image irradiance as functions of spatial coordinates. If we examine

the relationship in the spatial frequency domain, then from

$$\hat{E}(\bar{K}, f') = \iint d\bar{u} \hat{E}(\bar{u}, f') e^{-j2\pi\bar{K}\cdot\bar{u}} \quad (158)$$

and using (154) in (158)

$$\hat{E}(\bar{K}, f') = \iint d\bar{u} \iint d\bar{\xi} \hat{E}(\bar{\xi}, 0) \hat{S}(\bar{\xi} + \frac{d}{f'}, \bar{u}) e^{-j2\pi\bar{K}\cdot\bar{u}} \quad (159)$$

or with the following change of variable

$$\begin{aligned} \bar{\xi}' &= \bar{\xi} + \frac{d}{f'} \bar{u} \\ \bar{u} &= \frac{f'}{d} (\bar{\xi}' - \bar{\xi}) \\ d\bar{u} &= f'/d d\bar{\xi}' \end{aligned}$$

(159) becomes

$$\hat{E}(\bar{K}, f') = \frac{f'}{d} \iint d\bar{\xi}' \hat{S}(\bar{\xi}') e^{-j\frac{2\pi f' \bar{K} \cdot \bar{\xi}'}{d}} \iint d\bar{\xi} \hat{E}(\bar{\xi}, 0) e^{j\frac{2\pi f' \bar{K} \cdot \bar{\xi}}{d}} \quad (160)$$

Then from the Fourier transform definition (160) is given by

$$\boxed{\hat{E}(\bar{K}, f') = \frac{f'}{d} \hat{S}\left(\frac{\bar{K}f'}{d}\right) \hat{E}\left(-\frac{\bar{K}f'}{d}, 0\right)} \quad (161)$$

Spatial Frequency Domain

where: \bar{K} is a two dimensional spatial frequency vector (cycles/meter)

$\hat{S}(\bar{K})$ is the Fourier transform of the point spread function (eqn.157)

and is by definition the total unnormalized atmosphere-lens system optical transfer function (OTF).

The interpretation of equation (161) is as follows. If the object irradiance function is known (i.e. $\hat{E}(\bar{\xi}, 0)$) then the image irradiance function $\hat{E}(\bar{x}, f')$ is related to $\hat{E}(\bar{\xi}, 0)$ and the atmosphere-lens transfer function by

equation (161) in terms of the spatial frequency content of the object and image. The negative argument in the last term of (161) is a result of image inversion by the lens. Under the assumptions of circular symmetry for the lens and isoplanicity for the atmospheric path, the only effect of the negative sign is a reversal in reference direction for the image. The spatial frequency functions will be unaffected.

Examination of the inner integral in (157) yields

$$Q(\bar{\rho}) = \iint d\bar{R} w(\bar{R} + \frac{\bar{\rho}}{2}) w(\bar{R} - \frac{\bar{\rho}}{2}) \quad (162)$$

where the integral is over the lens aperture and (163) is shown to be [32]

$$Q(\bar{\rho}) = \frac{D_0^2}{2} \left[\cos^{-1} \left(\frac{|\bar{\rho}|}{D_0} \right) - \frac{|\bar{\rho}|}{D_0} \left(1 - \frac{|\bar{\rho}|^2}{D_0^2} \right)^{1/2} \right] \quad |\bar{\rho}| \leq D_0$$

$$= 0 \quad ; \quad |\bar{\rho}| > D_0 \quad (163)$$

The value of Q at $|\bar{\rho}| = 0$ is $\pi D_0^2/4$

so that by normalizing Q we have the lens modulation transfer function

$$M(\bar{\rho}) = \frac{4}{\pi D_0^2} Q(\bar{\rho}) \quad (164)$$

and equation (157) can be written as

$$\hat{S}(\bar{\xi}) = \frac{e^{-\alpha d}}{\lambda^4 d^2 f^2} \frac{\pi D_0^2}{4} \iint d\bar{\rho} M(\bar{\rho}) \text{MCF}(\bar{\rho}) e^{\frac{-j2\pi \bar{k}\bar{\rho} \cdot \bar{\xi}}{d}} \quad (165)$$

Now since the optical transfer function is the Fourier transform, $\hat{S}(\bar{K})$, of $\hat{S}(\bar{\xi})$ and letting

$$\bar{K} = \frac{k\bar{\rho}}{d} = \frac{\bar{\rho}}{\lambda d}$$

$$\bar{\rho} = \bar{K} \lambda d$$

$$d\bar{\rho} = \lambda d d\bar{K} \quad (166)$$

Using (166) in (165) gives

$$\hat{S}(\bar{\xi}) = \frac{\pi D_0^2 e^{-\alpha d}}{4\lambda^3 d f^2} \iint d\bar{K} M(\bar{K}\lambda d) \text{MCF}(\bar{K}\lambda d) e^{-j2\pi\bar{K}\cdot\bar{\xi}} \quad (167)$$

or

$$\text{OTF} = \hat{S}(\bar{K}) = \frac{\pi D_0^2 e^{-\alpha d}}{4\lambda^3 d f^2} M(\bar{K}\lambda d) \text{MCF}(\bar{K}\lambda d) \quad (168)$$

Since the system MTF is the normalized magnitude of the OTF we have finally that the atmosphere-lens system MTF is

$$\text{MTF} = \frac{M(K\lambda d) \text{MCF}(K\lambda d)}{\text{MCF}(0)} \quad (169)$$

Thus determination of the atmospheric MTF consists of determining the mutual coherence function, MCF.

C. Atmospheric Wave Structure Function

The approach given here is based on work by Fried [15] and considers a simple lens system illuminated by an atmospherically degraded plane wavefront. The result gives MTF for the system in terms of the atmospheric wave structure function.

Consider a simple imaging system as shown in Figure 7. Then for a wave incident on the lens which deviates little in phase and amplitude from a plane wave and designated as $\hat{\psi}_0(x,y,d)$ the resulting wave in the focal plane of the lens is shown to be [16] approximated by

$$\hat{\psi}(u,v,z) = A \iint dx dy \psi_0(x,y,d) e^{-\frac{j2\pi}{\lambda R} (xx' + yy')} \quad (170)$$

where λ is the optical wavelength

f is the lens focal length

and $k = \frac{1}{\lambda}$ is the wavenumber

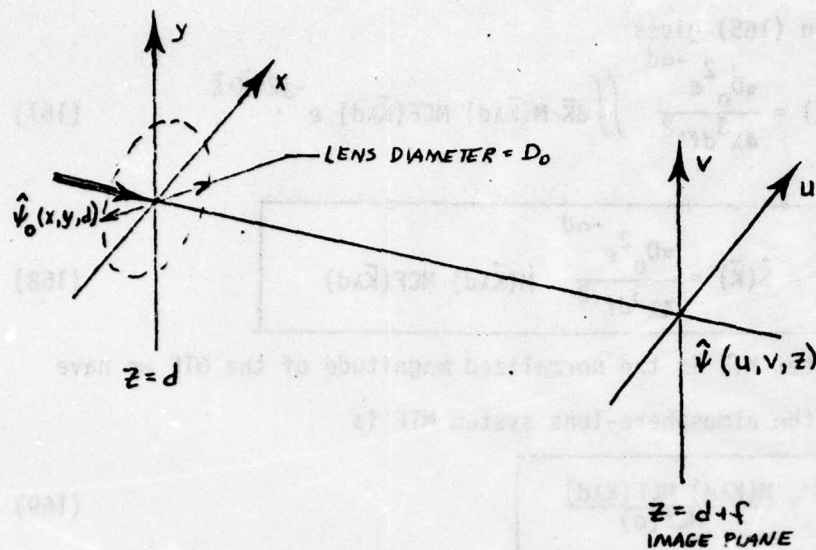


Figure 7. Focal plane field for a simple lens

If we use the shortened notation $\bar{\rho}' = (u, v)$ and $\bar{\rho} = (x, y)$ then (170) is

$$\hat{\psi}(\bar{\rho}', z) = A \iint d\bar{\rho}' \hat{\psi}_0(\bar{\rho}', 0) e^{-\frac{j2\pi k}{f}(\bar{\rho} \cdot \bar{\rho}')} \quad (171)$$

The MTF of the total system (optics plus atmosphere) is simply the normalized two-dimensional spatial Fourier transform of the intensity of the image. That is

$$\text{MTF}(\bar{K}) = B \iint d\bar{\rho} \hat{\psi}(\bar{\rho}') \hat{\psi}^*(\bar{\rho}') e^{2\pi j \bar{\rho}' \cdot \bar{K}} \quad (172)$$

where $\bar{K} = (\beta_x, \beta_y)$ spatial frequency in x and y directions

B is a normalizing constant which makes $\text{MTF}(0) = 1$.

Next, expressing the input wave as

$$\hat{\psi}(x, y, 0) = W(x, y, d) e^{[i\phi(w, y) + j\phi(x, y)]} \quad (173)$$

where $W(\bar{\rho}) = W(x, y, d) = 1; |\bar{\rho}| \leq D_0/2$
 $= 0; \text{ otherwise}$

is the lens pupil function. $\chi(\bar{\rho})$ and $\phi(\bar{\rho})$ are the log-amplitude and phase of the incident wave respectively. χ and ϕ will be random variables because of atmospheric refractive index fluctuations; therefore, $MTF(\bar{K})$ will be a random variable also. Thus we must describe the MTF in terms of statistical averages.

For the long-time exposure case Fried shows the average MTF to be

$$\langle MTF(\bar{K}) \rangle_{LE} = MTF_0(\bar{K}) e^{-\frac{1}{2} \hat{D}(\lambda f \bar{K})} \quad (174)$$

where $MTF_0(\bar{K})$ is the lens MTF
 $\hat{D}(\lambda f \bar{K})$ is the wave structure function for the atmospheric path and in terms of (173)

$$MTF_0(\bar{K}) = A^2 B \iint d\bar{\rho} W(\bar{\rho} - \lambda f \bar{K}) W(\bar{\rho})$$

$$D(\lambda f K) = [\phi(\bar{\rho}) - \phi(\bar{\rho} - \lambda f \bar{K})]^2 + [\chi(\bar{\rho}) - \chi(\bar{\rho} - \lambda f \bar{K})]^2.$$

For this case the effect of the atmosphere is separable from that of the lens and is given by:

$\langle MTF(\bar{K}') \rangle_{ATMOSPHERE} = e^{-\frac{1}{2} \hat{D}(\lambda \bar{K}')} \quad (175)$
LONG TIME EXPOSURE

where \bar{K}' has units of cycles per radian field-of-view; $\bar{K}' = f\bar{K}$. Thus (175) gives the MTF of the atmosphere in terms which are independent of the lens parameters.

For the short-exposure case the result is

$$\langle \text{MTF}(\bar{k}) \rangle_{\text{SE}} = \text{MTF}_0(\bar{k}) e^{-\frac{1}{2} \hat{D}(\lambda f \bar{k}) \left\{ 1 - \frac{1}{b} \left(\frac{\lambda f \bar{k}}{D_0} \right)^{1/3} \right\}} \quad (176)$$

where "b" is a constant between $\frac{1}{2}$ for the far-field and unity for the near field. In this case the effect due to the atmosphere cannot be expressed independently of the lens parameters, i.e.

$$\boxed{\langle \text{MTF}(\bar{k}') \rangle_{\text{ATMOSPHERE}} = \exp \left[-\frac{1}{2} \hat{D}(\lambda \bar{k}') \left\{ 1 - \frac{1}{b} \left(\frac{\lambda \bar{k}'}{D_0} \right)^{1/3} \right\} \right]} \quad (177)$$

SHORT EXPOSURE

In a later paper, Metheny and Philbrick [30] present an argument based on the Taylor "frozen flow" hypothesis for separating the optical transfer function for exposure times considerably shorter than those required for an ensemble average as was needed in (175).

Further insight into the results obtained in this section is gained from the following considerations. The wave structure function $\hat{D}(r)$ has been shown theoretically and experimentally [1] and [31] to have the relationship

$$\hat{D}(r) = A r^{5/3} = 6.88 \left(\frac{r}{r_0} \right)^{5/3} \quad (178)$$

where: $r_0 = (6.88/A)^{3/5}$ and the factor 6.88 is arbitrarily chosen to make the knee of the resolution function occur at $r_0 = D_0$ (see Fried [15] for a discussion). The factor "A" is a constant which depends on the propagation path length, the wavelength, the strength of the turbulence

and the nature of the unperturbed wavefront. Equation (178) has been shown to be valid for both plane and spherical wavefronts. Substituting (178) into (175) and (177) we have finally the MTF due to the atmosphere in terms of the parameter r_0 .

$$\langle \text{MTF}(\bar{k}') \rangle_{\substack{\text{LE} \\ \text{ATMOSPHERE}}} = e^{-3.44 \left(\frac{\lambda \bar{k}'}{r_0} \right)^{5/3}} \quad (179)$$

$$\langle \text{MTF}(\bar{k}') \rangle_{\substack{\text{SE} \\ \text{ATMOSPHERE}}} = e^{-3.44 \left(\frac{\lambda \bar{k}'}{r_0} \right)^{5/3} \left\{ 1 - \frac{1}{b} \left(\frac{\lambda \bar{k}'}{D_0} \right)^{1/3} \right\}} \quad (180)$$

D. Contrast Based on Em Theory

The total irradiance in the image plane $\bar{u} = (u, v)$ is the received object irradiance \hat{E} given by equation (154) plus ambient irradiance \hat{E}_a .

$$\hat{E}_{\text{TOT}} = \hat{E} + \hat{E}_a \quad (181)$$

From which we can define universal contrast at the image plane

$$C_{1,R} \triangleq \frac{\{\hat{E}_{\text{TOT}}\}_{\text{MAX}} - \{\hat{E}_{\text{TOT}}\}_{\text{MIN}}}{\{\hat{E}_{\text{TOT}}\}_{\text{MIN}}} \quad (182)$$

or

$$C_{1,R} \triangleq \frac{\hat{E}_{\text{MAX}} - \hat{E}_{\text{MIN}}}{\hat{E}_{\text{MIN}} + E_{a\text{MIN}}} \quad (183)$$

Then using (154) in (183) gives

$C_{1,R} = \frac{[\iint d\bar{\xi} \hat{E}(\bar{\xi}, 0) \hat{S}(\bar{\xi} + \frac{d}{f'} \bar{u})]_{\text{MAX}} - [\iint d\bar{\xi} \hat{E}(\bar{\xi}, 0) \hat{S}(\bar{\xi} + \frac{d}{f'} \bar{u})]_{\text{MIN}}}{[\iint d\bar{\xi} \hat{E}(\bar{\xi}, 0) \hat{S}(\bar{\xi} + \frac{d}{f'} \bar{u})]_{\text{MIN}} + \hat{E}_{a\text{MIN}}}$
IMAGE CONTRAST FOR ATMOSPHERE-LENS SYSTEM

(184)

Equation (184) gives the contrast in the image of an atmosphere-lens imaging system in terms of the atmosphere-lens point-spread function $\hat{S}(\bar{\xi})$, the ambient irradiance \hat{E}_a and the object irradiance function $\hat{E}(\bar{\xi}, 0)$. Inherent in the expression are dependence on atmospheric extinction and on object dimensions. Further analysis requires that $\hat{E}(\bar{\xi}, 0)$ be specified and that $\hat{S}(\bar{\xi})$ be known.

An alternate expression for $C_{1,R}$ is obtained in terms of spatial frequency functions

$$C_{1,R} = \frac{\left\{ \int_{-\infty}^{\infty} \hat{E}(\bar{K}, f') e^{j2\pi\bar{K} \cdot \bar{\xi}} d\bar{K} \right\}_{\text{MAX}} - \left\{ \int_{-\infty}^{\infty} \hat{E}(\bar{K}, f') e^{j2\pi\bar{K} \cdot \bar{\xi}} d\bar{K} \right\}_{\text{MIN}}}{\left\{ \int_{-\infty}^{\infty} \hat{E}(\bar{K}, f') e^{j2\pi\bar{K} \cdot \bar{\xi}} d\bar{K} \right\}_{\text{MIN}}} \quad (185)$$

where $\hat{E}(\bar{K}, f')$ is the image spatial transform as given by equations (161) and (168).

If $\hat{E}(\bar{\xi}, 0)$ is a one-dimensional periodic function such as would be represented by a bar chart (see Figure below) then

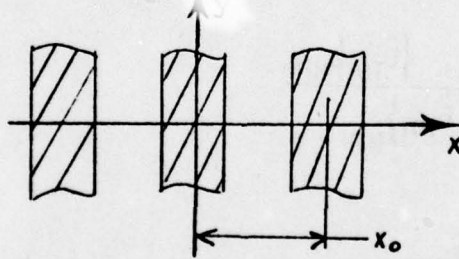


Figure 8. Bar pattern.

$$\hat{E}(\bar{K}, 0) = \sum_{n=-\infty}^{\infty} \hat{E}_n \delta(K_x - \frac{n}{x_0}, K_y) \quad (186)$$

where X_0 is the object period in the X-direction. The object is then represented by its Fourier series

$$\hat{E}(\bar{\xi}, 0) = \sum_{n=-\infty}^{\infty} \hat{E}_n e^{j2\pi n x / X_0} \quad (187)$$

and in the image plane from (187) and (161), for sinusoids

$$\hat{E}(\bar{\xi}, f') = \sum_{n=-\infty}^{\infty} \frac{f'}{d} \hat{S}\left(\frac{nf'}{X_0 d}\right) \hat{E}_n e^{j2\pi f' n x / d X_0} \quad (188)$$

Now if the contrast of the bars is computed using $x = 0$ as the maximum irradiance coordinate and $x = dX_0/2f'$ as the minimum irradiance coordinate in the image plane then

$$\hat{E}(\bar{\xi}, f')_{\text{MAX}} = \sum_{n=-\infty}^{\infty} \frac{f'}{d} \hat{S}\left(\frac{nf'}{X_0 d}\right) \hat{E}_n \quad (189)$$

and

$$\hat{E}(\bar{\xi}, f')_{\text{MIN}} = \sum_{n=-\infty}^{\infty} \frac{f'}{d} (-1)^n \hat{S}\left(\frac{nf'}{X_0 d}\right) \hat{E}_n$$

Then

$$C_{1,R} = \frac{2 \sum_{\substack{n=-\infty \\ n \text{ odd only}}}^{\infty} \hat{S}\left(\frac{nf'}{X_0 d}\right) \hat{E}_n}{\sum_{n=-\infty}^{\infty} \hat{S}\left(\frac{nf'}{X_0 d}\right) \hat{E}_n + \frac{d}{f'} \hat{E}_a} \quad (190)$$

UNIVERSAL CONTRAST PERIODIC BAR CHART
 THROUGH ATMOSPHERE-LENS

Equation (190) is the desired result and gives contrast in terms of the atmosphere-lens optical transfer function \hat{S} , the ambient irradiance \hat{E}_a and the Fourier coefficients, \hat{E}_n , for the bar pattern. The expression

also includes atmospheric absorption.

In terms of the modulation contrast, $C_{3,R}$, we have

$$C_{3,R} = \frac{\hat{E}_{MAX} - \hat{E}_{MIN}}{\hat{E}_{MAX} + \hat{E}_{MIN}} \quad (191)$$

Thus for the bar chart image

$$C_{3,R} = \frac{\sum_{\substack{n=-\infty \\ n \text{ odd only}}}^{\infty} \hat{S}\left(\frac{nf'}{x_0 d}\right) \hat{E}_n}{\sum_{\substack{n=-\infty \\ n \text{ even only}}}^{\infty} \hat{S}\left(\frac{nf'}{x_0 d}\right) \hat{E}_n + \frac{d}{2f'} E_a} \quad (192)$$

<p style="text-align: center;">MODULATION CONTRAST PERIODIC BAR CHART THROUGH ATMOSPHERE-LENS</p>

V. TEST AND MEASUREMENT METHODS

A. Visibility and Contrast

For the contrast reduction expression

$$\frac{C_R}{C_0} = \frac{N_0'}{N_R'} e^{-\sigma_0 \bar{R}} \quad (193)$$

complete specification requires measurement of the background radiances N_0' and N_R' at ranges of 0 and R meters in addition to "visibility" measurements. Since the visibility range R_v is defined to be the range at which contrast is reduced to 2 percent of C_0 , then R_v lends itself well to measurement with a telephotometer. Using measured values of R_v , N_0' and N_R' , we can compute the value of the extinction coefficient, i.e.

$$\sigma_0 = \frac{1}{R_v} \ln \left(.02 \frac{N_R'}{N_0'} \right) \quad (194)$$

Once σ_0 is known, contrast measurements may be taken at various ranges and compared with the predicted values of (193). The comparison is in fact a comparison of \bar{R} the optical slant range with R, the true slant range.

If the measurement situation is for a horizontal path with an object being measured against a sky background then $N_R' = N_0'$ and it is usually

assumed that $\bar{R} = R$ so that measurement of contrast versus range will give all the pertinent information. The result should be as shown in Figure 9. Extinction coefficient is then easily obtained from the slope of the curve while visibility range is that range at which $C_R/C_0 = .02$.

For the case of (193) where the path is not horizontal against a sky background, the effects of N_R' and \bar{R} must be considered. Since

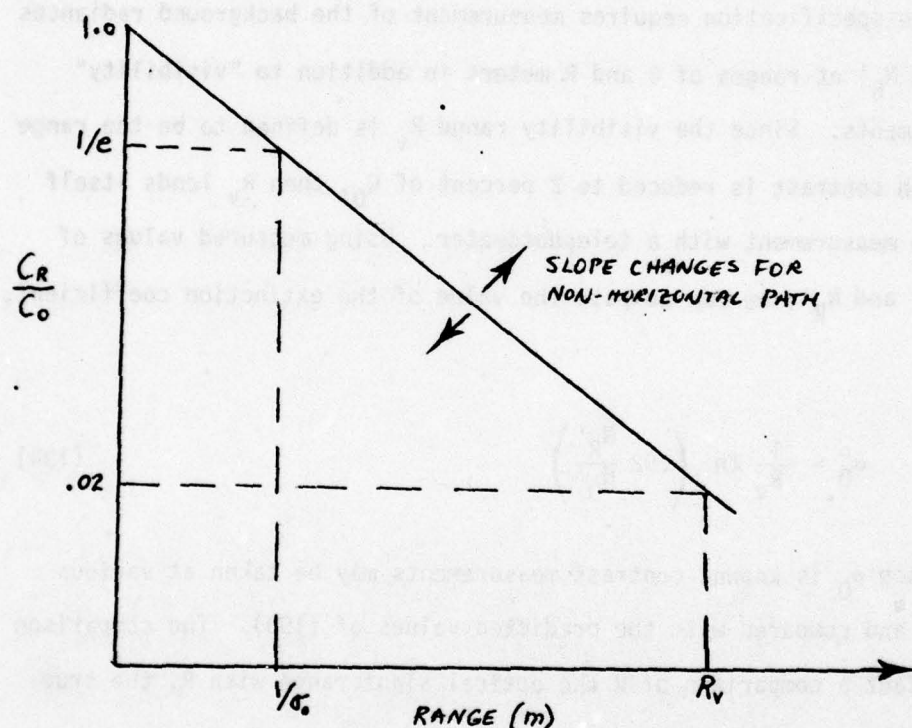


Figure 9. Visibility and extinction coefficients from contrast measurements.

N_R' is the background radiance at range R and since all objects (bright or dark) tend to approach the air light radiance at long ranges, then

N_R' will decrease with R for a bright background and increase with R for a dark background. Thus an object against a background can have its contrast reduced either more rapidly or less rapidly for the sky-to-ground case than for the horizontal case. With respect to Figure 9, this effect would be realized as a change in slope of the curve. In fact, it is not unreasonable to expect that small nonlinearities may exist in the N_R' dependence on R . The optical slant range \bar{R} is expected to be a gradually decreasing function of R since it is a measure of the optical path which depends on refractive index. The effect of Figure 9 is to rotate the curve slightly counter-clockwise about $C_R/C_0 = 1.0$.

A number of precautions and effects must be considered in performing the indicated measurements. First, since any measuring instrument will be dependent on spatial frequency content of a test target, this dependency must be known and compensated in interpreting the measurement data. This problem, of course, arises because of the size with range variation of what the measurement instrument sees. Otherwise some zoom mechanism must be used in conjunction with the instrument to maintain constant relative image size. If such an arrangement is used, however, it is still necessary to consider the MTF of the zoom optics and correct for it.

B. Wave Structure Function and Atmospheric Mutual Coherence Function

MTF effects of the atmosphere were shown in Section 4.3 to depend on a quantity $\hat{D}(r)$ which was called the wave structure function where " r " is a correlation parameter representing spatial separation. It has been shown for a large range of values " r " that the wave structure function is

adequately represented by [33]

$$\hat{D}(r) = C_n^2 r^{2/3} \quad (195)$$

where C_n^2 is the refractive index structure constant, or more accurately, the refractive index structure function, since C_n^2 is not a constant, but varies with time and space. In fact, as seen by curves in [34], C_n^2 can vary over several orders of magnitude during a typical day. It is strongly dependent on the time of day as well as altitude. The wave structure function can be written as

$$\hat{D}(r) = \hat{D}_A(r) + \hat{D}_S(r) \quad (196)$$

where: $\hat{D}_A(r)$ is the variance of the amplitude difference fluctuations of the refractive index between two points spaced "r" apart and $\hat{D}_S(r)$ is the similar variance of the phase terms. Although $\hat{D}_A(r)$ may be measured using a pair of photodetectors [1] and $\hat{D}_S(r)$ may be measured with an interferometer [35] the most commonly cited technique is to measure a function C_T^2 which is defined as the temperature structure function using temperature profile measurements. From C_T^2 the refractive index structure function C_n^2 is calculated which is then related to the MTF of the atmosphere.

For optical frequencies a good approximation for the refractive index "n" is given by [36]

$$n = 1 + A(p/T) \left[1 + B \frac{e}{pT} \right] \quad (197)$$

where p = pressure (millibars)

T = temperature (Kelvin)

e = vapor pressure (millibars)

B is a constant, and

$$A \approx 77.5(10^{-6})[1 + 5.15(10^{-3})\lambda^{-2} + 1.07(10^{-4})\lambda^{-4}] \quad (198)$$

is a wavelength dependent constant. Higher order terms in (197) are neglected. It can be shown that if humidity effects can be ignored then

$$C_n^2 \sim \frac{A_p}{T^2} C_T^2 \quad (199)$$

where $C_T^2 r^{2/3} = \langle [T(r_1) - T(r_2)]^2 \rangle \quad (200)$

$$r = |r_1 - r_2|$$

Examples of C_T^2 for various temperature measurement conditions (eg. altitude, time of day, etc.) are given in [37]. Example MTF curves are given in the same reference for various viewing conditions.

The atmosphere-lens imaging system is specified by either its point spread function (PSF) equation (157) or its modulation transfer function (MTF), $\hat{S}(\frac{\bar{K}f'}{d})$ in equation (161).

From (161) we can solve for the total system MTF

$$\hat{S}(\frac{\bar{K}f'}{d}) = \frac{d}{f} \frac{\hat{E}(\bar{K}, f')}{\hat{E}(-\frac{\bar{K}, f'}{d}, 0)} \quad (201)$$

where $\hat{E}(\bar{K}, f')$ is the two-dimensional spatial fourier transform of the output image irradiance function. This quantity can be computed digitally

from a measurement of the two-dimensional irradiance function in the image plane of a measurement instrument. $\hat{E}\left(-\frac{\bar{K}f'}{d}, 0\right)$ is the scaled two-dimensional spatial fourier transform of the input object irradiance. It can be measured or computed from knowledge of the object to be imaged (eg. a bar chart pattern). The effect of ambient irradiance or average irradiance level of the bar chart is on only the $\bar{K} = 0$ term for equation (201). Thus, these effects can be quantized and accounted for. The computation and scaling for the input spatial transform is best done digitally. Further to avoid overflow problems, the result in (193) should be computed only for values of \bar{K} such that the denominator is larger than zero, and in fact, larger than some system noise level.

The method outlined above should give reasonable accuracy for the atmosphere-lens system at a given range. Measurements at different ranges should affect the MTF. In particular, the absorption term in equation (157) varies exponentially with range "d". Further, the statistical characteristics of the mutual coherence function may change with "d". The complete dependence on "d" is difficult to predict analytically. However, a measurement program which includes several ranges within a total expected range variation could provide data for experimental prediction of the total range effect.

Essentially the experimental program should consist of measurements of image plane irradiance for computation of equation (201). The same bar chart target can be used at the different ranges; however, care must be exercised so that the received image size is not so small as to be down in the noise of the measuring system MTF. An instrument with adjustable field-of view should be capable of compensating for target size variations

over a typical range variation of one to five kilometers.

For determining contrast transfer through the atmosphere lens system, equations (184) and (185) give the theoretical results. A result more amenable to experimental varification, however, is to use equation (182) where the actual image irradiance levels can be measured. This result can be compared with the inherent object irradiance to get contrast transfer through the system.

The effect of target size on contrast transfer is easily measured by including bar charts of varying size at each range. Care must be taken to correct for the lens effect if it is desired to know the effect of the atmospheric path only.

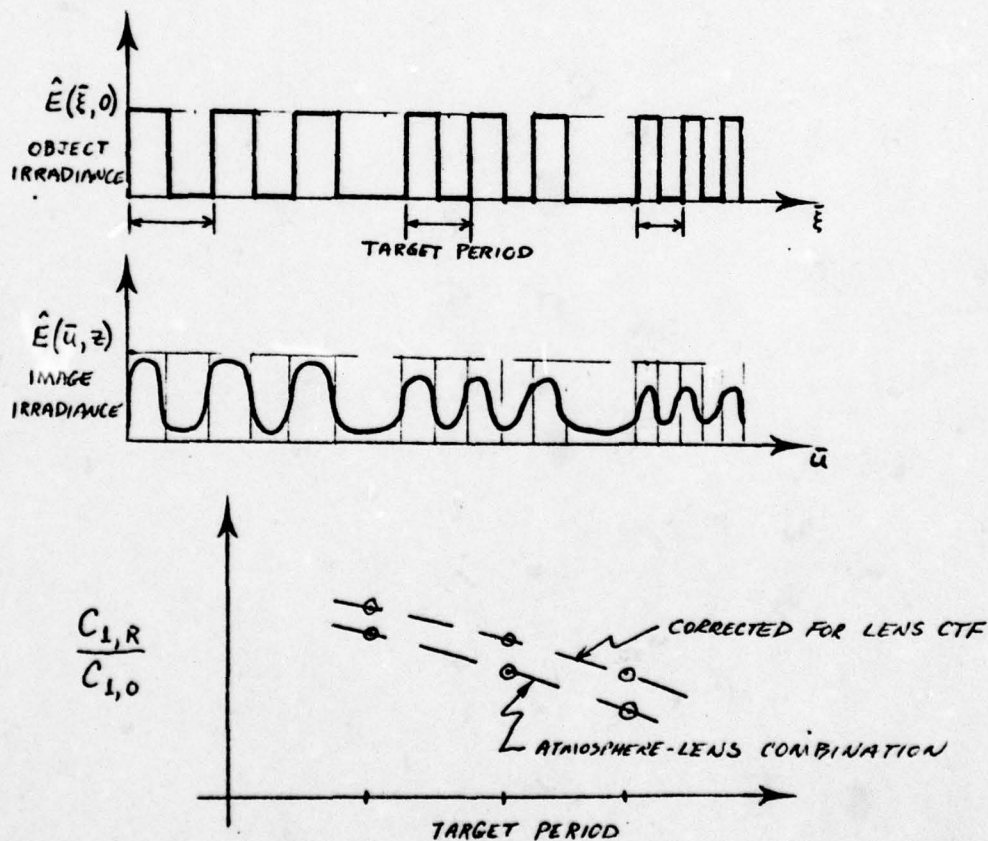


Figure 10. Contrast transfer measurement.

In summary, the methods for measuring effects of the atmosphere on an optical system can be as complex as the theory which predicts these effects. A more detailed study of this topic should be done to fully investigate the validity of specific measurement techniques in particular as related to theoretical results. Only after evaluation of such an effort would it be practical to investigate simulation methods.

VI. SUMMARY AND CONCLUSIONS

In evaluating the usefulness of the theoretical expressions developed in this report in terms of their applicability for determining performance of imaging seekers it becomes necessary to examine the meaning of various assumptions and approximations made during the course of those theoretical developments. For example, some method for reconciling a monochromatic theory with a wideband, spectrally non-trivial application such as is true for the visible and near-infrared imaging seeker must be found. This problem has not been treated rigorously in the literature. The contrast theory as developed by Duntley and Middleton has been extended to give results for a finite wavelength band and was shown to have little effect except for slant ranges greater than or close to the meteorological range. Under these conditions there is of course serious doubt that the sensors can operate at all.

If we now examine the implications of a finite spectral band on the results of Section 4.0, then the MTF of the atmosphere for imaging seeker applications should be the result. It is between equations (75) and (76) that a monochromatic field was assumed. The major advantage of this assumption was that it allowed separation of the time and spatial dependence of the field. Therefore, a solution $\hat{\phi}(x,y,z,v)$ which is not necessarily monochromatic should still satisfy the scalar Helmholtz wave equation. Unfortunately the approximate solution methods take advantage of the separability and in some cases assume a propagating

plane wave so that these methods are not easily extended to the case of a continuous spectral band and a general $\hat{\phi}(x,y,z,v)$.

If a monochromatic wave $\hat{\phi}(x,y,z,v_0)$ satisfies the wave equation, then a summation of monochromatic waves should also satisfy the wave equation, i.e.

$$\hat{\phi} = \sum_i \hat{\phi}_i(x,y,z,v_i) \quad (202)$$

and if we extend this concept over a continuous range of optical frequencies we get as a solution also

$$\hat{\psi} = \int_{\Delta v} \hat{\phi}(x,y,z,v) dv \quad (203)$$

Further development and evaluation of this approach are needed. A different approach based on the theory of partial coherence is another alternative for the prediction of broadband MTF effects of the atmosphere.

The development given in this section for including broadband effects on the contrast transfer function is easily applied to the imaging sensor application. The development given for broadband effects on the MTF due to atmospheric turbulence requires further work.

In conclusion, further details of the experimental program need to be worked out. Additionally, there are still some rough spots in the theoretical development, which if solved, could provide more useful analytical predictions of the effects of range and target size on MTF and contrast transfer.

The comprehensive rigorous theory which was developed in this effort lays the groundwork for a more complete description of the finite spectral band atmospheric imaging system.

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APPENDIX A: VISIBILITY, VISIBILITY RANGE AND METEOROLOGICAL RANGE

Visibility, visibility range and meteorological range are three terms for the same quantity which is defined specifically as the range at which the apparent contrast of an object is two percent of its inherent contrast. Thus "visibility" has units of distance and is a somewhat arbitrary measure of how "clear" the atmosphere is on any given day or for a given set of meteorological conditions. For the simplest contrast transfer relationship given in equation (12) if R_v is the visibility range then

$$\frac{C_R}{C_0} = .02 = e^{-\sigma_0 R_v} \quad (A-1)$$

or $-\sigma_0 R_v = \ln .02 = -3.912$

$$R_v = \frac{3.912}{\sigma_0} \quad (A-2)$$

or using $\sigma_0 = \frac{3.912}{R_v}$ the contrast at range R is given by

$$\frac{C_R}{C_0} = e^{-\frac{3.912R}{R_v}} \quad (A-3)$$

For a "standard clear atmosphere" $R_v = 23.5$ km which implies that σ_0 for a standard clear atmosphere is $.1665 \text{ km}^{-1}$.

Now if we adhere rigorously to the definition for visibility given

above then the sky-to-ground observer situation gives a visibility

$$\bar{R}_V = \frac{-1}{\sigma_0} \ln \left(.02 \frac{N_{R'}}{N_{O'}} \right) = \frac{1}{\sigma_0} \ln \left(50 \frac{N_{O'}}{N_{R'}} \right) \quad (A-4)$$

where \bar{R}_V is the optical slant visibility range and is dependent on the background radiance as well as $f(r)$.

The simpler definition above is typically the one used for visibility.

APPENDIX B: CONTRAST DEFINITIONS

A number of definitions for contrast are found in the literature of which three commonly used definitions are given below. Judgement as to which is best is left to the discretion of the user; however, the significant differences are identified.

$$C_1 \triangleq \frac{L_0 - L_b}{L_b} = \frac{L_0}{L_b} - 1; -1 \leq C_1 \leq \infty \quad (\text{Universal})$$

$$C_2 \triangleq \frac{|L_0 - L_b|}{L_b} = \left| \frac{L_0}{L_b} - 1 \right|; 0 \leq C_2 \leq \infty \quad (\text{Television})$$

$$C_3 \triangleq \frac{(L_0 - L_b)}{(L_0 + L_b)} = \frac{\frac{L_0}{L_b} - 1}{\frac{L_0}{L_b} + 1}; -1 \leq C_3 \leq 1 \quad (\text{Modulation})$$

Definition C_1 has the property that for any given background radiance L_b , the contrast increases linearly as the object radiance increases. It takes on values of -1 for a totally dark object to $+\infty$ for a totally dark background. Definition C_2 is similar to C_1 except that the absolute magnitude sign eliminates any negative contrast values. The argument is that the important thing is radiance difference between object and background and not which is brighter. Definition C_3 eliminates large numbers and maps contrast into the range -1 to $+1$; however, the contrast dependence on increasing object radiance is now a non-linear function. All three contrast definitions are shown plotted ⁱⁿ Figure B-1 versus L_0/L_b .

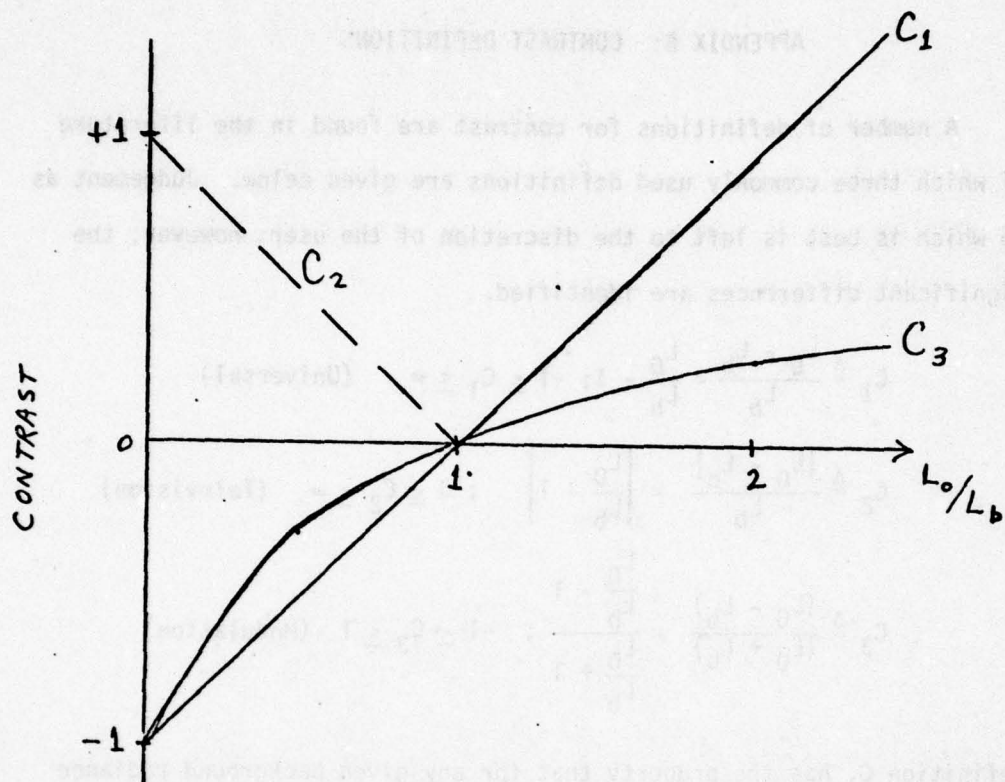


Figure B-1. Contrast definitions.

APPENDIX C: LINEAR SYSTEMS, POINT SPREAD FUNCTION, MODULATION TRANSFER FUNCTION AND CONTRAST TRANSFER FUNCTION

Based on assumptions of a weakly inhomogeneous, weakly absorbing atmosphere with enough uniformity that the target/imaging system can be considered to be in an isoplanatic region (i.e., the atmospheric region of interest can be assumed to be statistically shift-invariant) and taking into account the size scaling and reversal of images from a simple lens system, the atmosphere-lens system is a quasi-linear, shift-invariant system and can be described in the following way.

If the object to be imaged is represented by its complex monochromatic spatial amplitude function $\hat{\psi}(x,y,0)$, then the image spatial amplitude function is given by the convolution of $g(x,y,z)$ with $\hat{\psi}(x,y,0)$.

$$\hat{\psi}(x,y,z) = g(x,y,z) * \hat{\psi}(x,y,0) \quad (C1)$$

where $g(x,y,z)$ is the two-dimensional green's function for the atmosphere-lens combination and $z = d + f'$ is the total object-image separation.

See Figure C1.

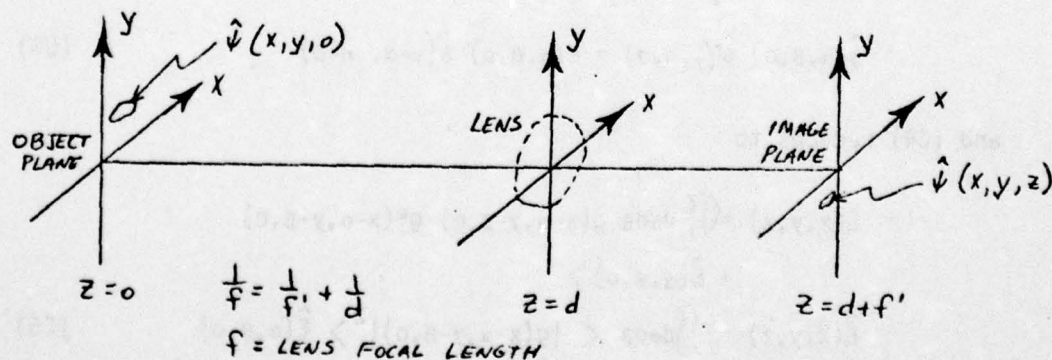


Figure C-1. Atmospheric-lens imaging system.

Or schematically, equation C1 can be represented by Figure C2

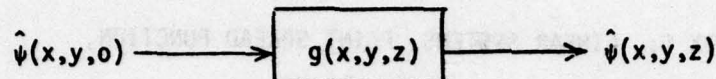


Figure C-2. Schematic representative, linear system.

Our interest is in the real image intensity and not in the complex spatial amplitude function. So we have the image irradiance function of spatial coordinates

$$\hat{E}(x,y,z) = \langle \hat{\psi}(x,y,z) \hat{\psi}^*(x,y,z) \rangle \quad (C2)$$

where the brackets denote an ensemble average and are necessary because the atmospheric green's function is a random quantity.

From (C1) and (C2)

$$\hat{E}(x,y,z) = \langle \{g(x,y,z) * \hat{\psi}(x,y,0)\} \cdot \{g^*(x,y,z) * \hat{\psi}^*(x,y,0)\} \rangle \quad (C3)$$

where the superscript * denotes complex conjugate. Or

$$\begin{aligned} E(x,y,z) = & \left\langle \left(\iiint d\alpha d\beta d\gamma d\eta \ g(x-\alpha, y-\beta, z) \hat{\psi}(\alpha, \beta, 0) \right. \right. \\ & \left. \left. g^*(x-\gamma, y-\eta, z) \hat{\psi}^*(\gamma, \eta, 0) \right) \right\rangle \end{aligned} \quad (C4)$$

If the source is spatially incoherent then

$$\hat{\psi}(\alpha, \beta, 0) \hat{\psi}^*(\gamma, \eta, 0) = \hat{E}(\alpha, \beta, 0) \delta(\gamma-\alpha, \eta-\beta) \quad (C5)$$

and (C4) reduces to

$$\begin{aligned} \hat{E}(x,y,z) = & \left\langle \left(\iint d\alpha d\beta \ g(x-\alpha, y-\beta, 0) \ g^*(x-\alpha, y-\beta, 0) \right. \right. \\ & \left. \left. \cdot \hat{E}(\alpha, \beta, 0) \right) \right\rangle \\ \hat{E}(x,y,z) = & \iint d\alpha d\beta \ < |g(x-\alpha, y-\beta, 0)|^2 > \hat{E}(\alpha, \beta, 0) \end{aligned} \quad (C6)$$

Equation (C6) expresses the image plane irradiance $\hat{E}(x,y,z)$ in terms of the object plane irradiance $\hat{E}(\alpha,\beta,0)$ and the mean square value of the atmosphere-lens green's function. Can also write (C7) as

$$\hat{E}(x,y,z) = |g(x,y,z)|^2 * \hat{E}(x,y,0) \quad (C7)$$

or

$$\hat{E}(x,y,z) = s(x,y,z) * \hat{E}(x,y,0) \quad (C8)$$

where: $s(x,y,z) = |g(x,y,z)|^2$ is the atmosphere-lens point spread function. That is $s(x,y,z)$ is the two dimensional image irradiance function at $z = z$ for an ideal point source input irradiance function at $z = 0$.

From the convolution theorem and taking the spatial Fourier transform of (A8) we get

$$\hat{E}(k_x, k_y, z) = \mathcal{F}\{s(x,y,z)\} \hat{E}(k_x, k_y, 0) \quad (C9)$$

where $E(k_x, k_y, 0)$ is the input irradiance spatial frequency function

$E(k_x, k_y, z)$ is the output image irradiance spation frequency function

and k_x, k_y are spatial frequencies in x and y direction respectively. Further

$$\mathcal{F}\{s(x,y,z)\} = S(k_x, k_y, z) \quad (C10)$$

where $S(k_x, k_y, z)$ is the combined atmosphere-lens optical transfer function for monochromatic radiation.

The modulation transfer function for the combined system is then

$$MTF(k_x, k_y, z) = \frac{|S(k_x, k_y, z)|}{|S(0, 0, z)|} \quad (C11)$$

The MTF as expressed in (C11) gives the total system normalized response to sinusoidal spatial irradiance as a function of spatial frequency.

This result may be compared to the contrast transfer function (CTF) in which case CTF is the normalized total system response to a square-wave spatial irradiance as a function of spatial frequency.

In terms of equations (C8) and (C9), to get MTF the input irradiance function is a pure sinusoid, i.e.,

$$\hat{E}(x,y,o) = \frac{\hat{E}_0}{2} (1 + \cos 2\pi\{k_{o,x} x + k_{o,y} y\}) \quad (C12)$$

or

$$\begin{aligned} \hat{E}(k_x,k_y,o) = & \frac{\hat{E}_0}{2} \delta(k_x,k_y) + \frac{\hat{E}_0}{4} \delta(k_x-k_{o,x},k_y-k_{o,y}) \\ & + \frac{\hat{E}_0}{4} \delta(k_x+k_{o,x},k_y+k_{o,y}) \end{aligned} \quad (C13)$$

To get CTF, the input irradiance function is

$$\hat{E}(x,y,o) = SQ(x,y,o) \quad (C14)$$

where $SQ(x,y,o)$ is a two dimensional square wave as shown in Figure C3.

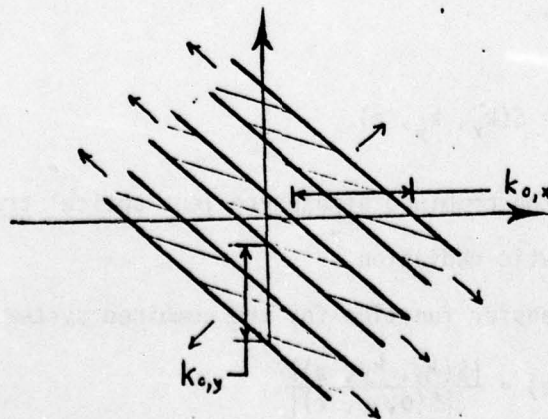


Figure C-3. General two-dimensional bar pattern.

The two dimensional Fourier spatial transform of (C14) gives

$$\hat{E}(k_x, k_y, 0) = \sum_{m=-\infty}^{\infty} A_m \delta(k_x - mk_{0,x}, k_y - mk_{0,y}) \quad (C15)$$

where the coefficients A_m are given by

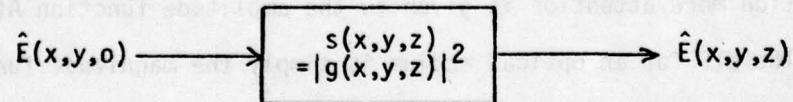
$$A_m = \frac{1}{2} \text{sinc}\left(\frac{m}{2}\right) \quad (C16)$$

$$\text{and sinc } \triangleq \frac{\sin \pi x}{\pi x} \quad (C17)$$

Thus the square wave input irradiance function consists of a summation of weighted sinusoidal spatial irradiance functions. Then it follows that the response to a square wave input is the sum of the responses to the weighted sinusoids comprising the square wave. Then we have the following relationship between CTF and MTF.

$$\text{CTF}(k_x, k_y, z) = \sum_m A_m \text{MTF}(k_{x,m}, k_{y,m}, z) \quad (C18)$$

So in terms of the real image irradiance functions the following linear system concept applies.



The remainder of this Appendix gives a different approach to definition of MTF and gives some useful relationships among several optical system performance functions.

Analogous to the transfer function, $H(f)$, of an electrical network, we can define an optical transfer function (OTF) which relates input and output spatial frequency content for an optical system. The OTF is then given by

$$\text{OTF} = \frac{I(k_x, k_y)}{O(k_x, k_y)} = A(k_x, k_y) e^{j\theta(k_x, k_y)} \quad (C19)$$

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where $I(k_x, k_y)$ = image plane (output) representation as a function
of spatial frequencies k_x and k_y
 $O(k_x, k_y)$ = object plane (input) representation.

The OTF is in general complex so that $A(k_x, k_y)$ is its amplitude and $\theta(k_x, k_y)$ is its phase. Both are functions of spatial frequency. There is also an implicit dependence on spectral content of the radiation.

The computation and/or measurement of the optical transfer function is accomplished as a time-averaged value since all practical photosensors have finite response times and are in fact sensitive to the optical intensity, not the amplitude. However, for measuring the OTF of a slowly time-varying medium such as the atmosphere, the observation time becomes an important parameter. In other words, any observation will include a large number of optical wavelengths over which the effect is averaged, but the given wavelength train can be modulated by the properties of the atmosphere at rates which are measurable.

Since the spatial phase function is essentially random for incoherent radiation more attention is given to the amplitude function $A(k_x, k_y)$.

The MTF for an optical medium is simply the magnitude function $A(k_x, k_y)$ normalized to give a value of unity at $k_x = k_y = 0$, where the object signal is a spatial sine wave. Definitions for MTF and related functions are given below.

Modulation Transfer Function (MTF) - a spatial frequency function which gives the normalized sine wave response of an optical element.

Point Spread Function (PSF) - a spatial function which describes

how an optical element spreads or distorts an ideal point source. May be defined as the normalized intensity distribution in the image plane of an ideal point source in the object plane.

Edge Function (EF) - the normalized response of an optical element to an optical step function (edge). EF is a spatial function.

Contrast Transfer Function (CTF) - a function of spatial frequency which gives the normalized response of an optical element to a square-wave input.

Properties of the Above Functions

- a. $0 \leq \text{MTF} \leq 1$
- b. $0 \leq \text{CTF} \leq 1$
- c. MTF of system elements can be cascaded to give system MTF.
- d. CTF cannot be cascaded
- e. CTF easier to measure since optical square wave is easier to generate than optical sine wave.
- f. CTF can be converted to MTF.
- g. EF is the integral of PSF.
- h. MTF and PSF form a Fourier transform pair.
- i. EF easier to measure than PSF since ideal point source is impossible to model.

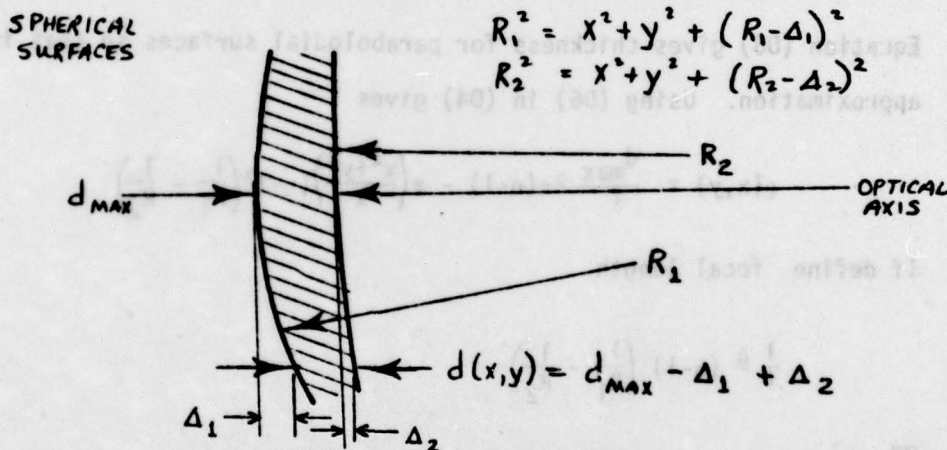
APPENDIX D: TRANSMISSION FUNCTION FOR A THIN LENS

If $\hat{t}(x,y)$ is the lens transmission function assume

$$\hat{t}(x,y) = \hat{p}(x,y) e^{j\phi(x,y)} \quad (D1)$$

where: $\hat{p}(x,y)$ is the pupil function and represents the aperture without the lens.

$\phi(x,y)$ is the phase function which depends on the optical thickness of the lens.



$$d(x,y) = d_{MAX} - [R_1 - (R_1^2 - x^2 - y^2)^{1/2}] + [R_2 - (R_2^2 - x^2 - y^2)^{1/2}] \quad (D2)$$

Then the total phase delay through the lens is

$$\hat{\phi} = 2\pi k n d(x,y) = 2\pi k d(x,y) + \phi(x,y) \quad (D3)$$

Now the additional phase delay through the lens is (over what would occur without lens)

$$\phi(x,y) = 2\pi k (\eta - 1) d(x,y) \quad (D4)$$

where: $d(x,y)$ is given in (D2) and, as can be checked in optics references, we have

$$\text{optical path length} = (\text{refractive index} \cdot \text{actual length})$$

Approximation: for a "thin" lens

$$\begin{aligned} x^2 + y^2 &\ll R_1^2 \\ x^2 + y^2 &\ll R_2^2 \end{aligned} \quad (D5)$$

So

$$d(x,y) \approx d_{\max} - \frac{x^2 + y^2}{2} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (D6)$$

Equation (D6) gives thickness for paraboloidal surfaces so that is the approximation. Using (D6) in (D4) gives

$$\phi(x,y) \approx \frac{d_{\max}}{\lambda} 2\pi(n-1) - \pi \left(\frac{x^2 + y^2}{\lambda} \right) (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (D7)$$

If define focal length

$$\frac{1}{f} \triangleq (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (D8)$$

or

$$f = \frac{1}{n-1} \left(\frac{R_1 R_2}{R_2 - R_1} \right)$$

Then the lens transmission function is

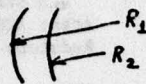
$\hat{t}(x,y) = \hat{p}(x,y) e^{-j (x^2 + y^2) / \lambda f}$	(D9)
THIN LENS TRANSMISSION FCN	

where $\hat{p}(x,y)$ is the pupil function.

CONVENTION ON LENS RADII AND FOCAL LENGTH

$$f = \frac{1}{n-1} \left(\frac{R_1 R_2}{R_2 - R_1} \right) \quad (D10)$$

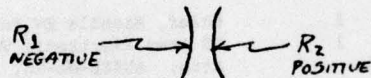
where both R_1 and R_2 were convex surfaces to left and considered positive



for surfaces concave to left, the radii should be negative



The conditions under which the focal length is negative can be found by examining eqn. (D10) eg.



Equation (D9) is the desired result.

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